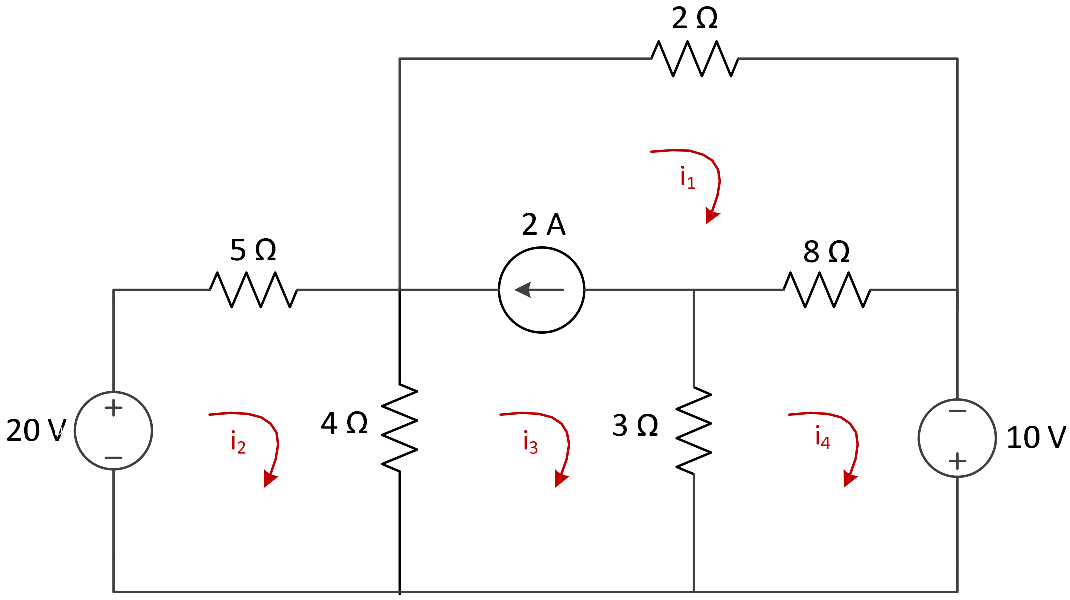
Q1 - a (15 points)



Write the mesh equations for i_1, i_2, i_3, and i_4 in the matrix form

$$\begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

(Note: Don't solve for the mesh currents i_1, i_2, i_3, and i_4)

Solution

From the super-mesh

$2i_1 + 8(i_1 - i_4) + 3(i_3 - i_4) + 4(i_3 - i_2) = 0$

$10i_1 - 4i_2 + 7i_3 - 11i_4 = 0$ ----*(1)*

$i_1 - i_3 = 2$ ----- *(2)*

Mesh-2

$-20 + 5i_2 + 4(i_2 - i_3) = 0$

$9i_2 + -4i_3 = 20$ ----*(3)*

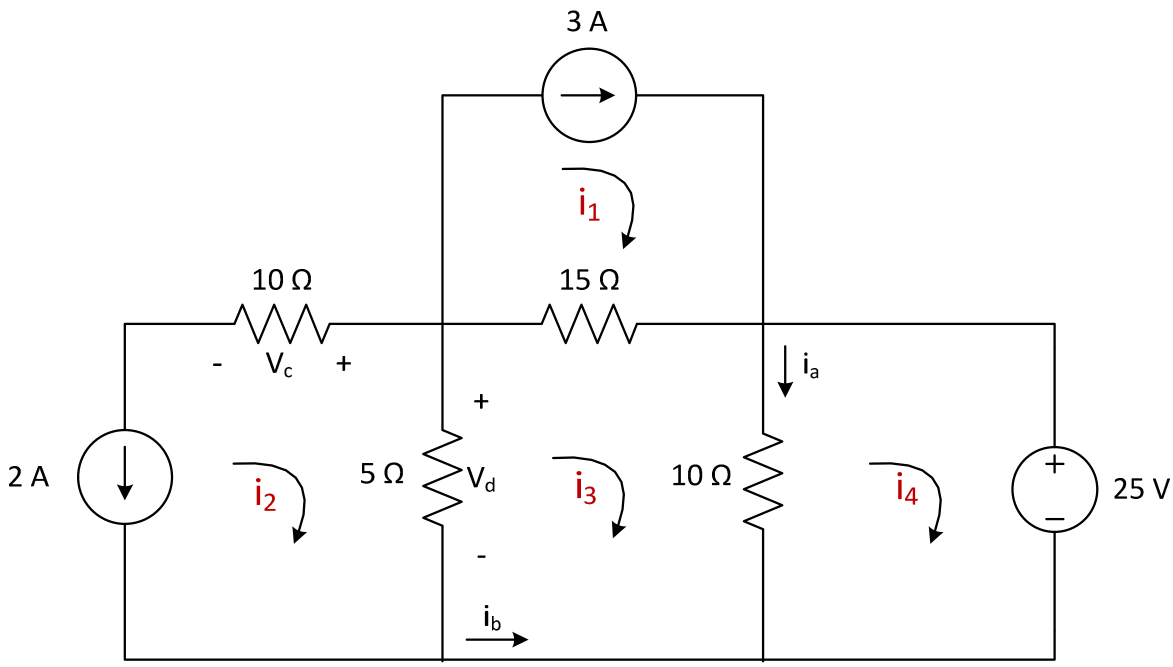
Mesh-4

$-10 + 3(i_4 - i_3) + 8(i_4 - i_1) = 0$

$-8i_1 - 3i_3 + 11i_4 = 10$ ----*(4)*

$$\begin{bmatrix} 10 & -4 & 7 & -11 \\ 1 & 0 & -1 & 0 \\ 0 & 9 & -4 & 0 \\ -8 & 0 & -3 & 11 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 20 \\ 10 \end{bmatrix}$$

Q1 – b (15points)



Given that

$$i_3 = 0.5\ A$$
$$i_4 = -2\ A$$

a) Solve for i_1 and i_2
b) Calculate the branch currents (i_a and i_b)
c) Calculate the voltages (V_c and V_d)
d) Calculate the power delivered by the 3 A current source.

Solution

a)

$i_1 = 3$ *A*

$i_2 = -2$ *A*

b)

$i_a = i_3 - i_4 = 0.5 + 2 = 2.5$ *A*

$i_b = i_d + 2 = (i_2 - i_3) + 2 = (-2-0.5)+2 = -0.5$ *A*

c)

$V_c = -10\ (\ i_2\ ) = -10\ (\ -2) = 20$ *V*

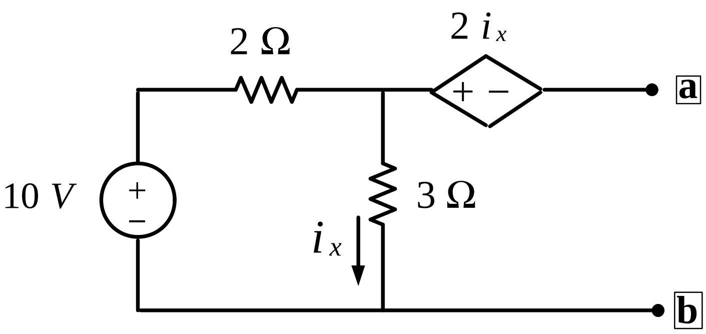
$V_d = 5\ (i_2 - i_3) = 5(-2-0.5) = -12.5$ *V*

d)

$V_{15\Omega} = 15(i_1 - i_3) = 15(3-0.5) = 15(2.5) = 37.5$ *V*)

$P_{3A} = -(3)(37.5) = -\ 112.5$ *W*

Q2 (20)



For the circuit shown above , find the followings :

(a) The open circuit voltage between terminals **a** and **b** ?

(b) The short circuit current through the terminals **a** and **b** ?

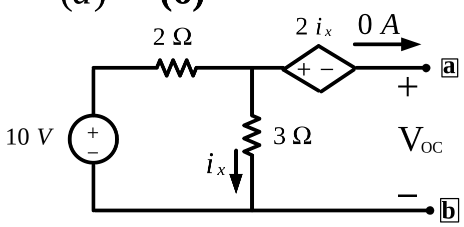
(c) The Thevenin equivalent resistant between terminals **a** and **b** ?

(d) The load resistant R_L between terminals **a** and **b** that will absorb the maximum power ?

(e) The maximum power absorbed by the load resistant R_L in part (d) ?

Solution

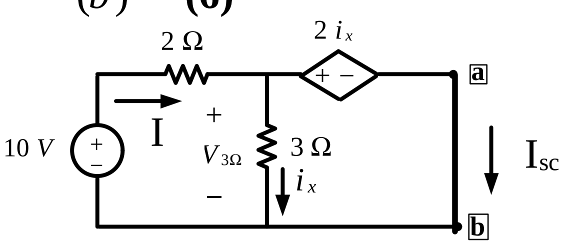
(a) **(6)**



KVL on loop 1 $-10+5i_x = 0 \Rightarrow i_x = 2A$

KVL on loop 2 $-3i_x + 2i_x + V_{OC} = 0 \Rightarrow V_{OC} = 2\text{ V}$

(b) **(6)**



$$V_{3\Omega} = 3i_x = 2i_x \Rightarrow i_x = 0$$

$$I_{SC} = I = \frac{10}{2} = 5A$$

(c)

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{2}{5}\ \Omega$$

(d)

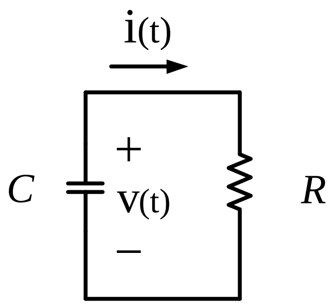
$$R_L = R_{TH} = \frac{2}{5}\ \Omega$$

(d)

$$P_{max} = \frac{V^2_{oc}}{4R_{TH}} = \frac{4}{4(2/5)} = \frac{5}{2} = 2.5\ \text{W}$$

Q3 (25)

Part I For the RC circuit shown below , the voltage across the capacitor and the current through the capacitor are



$v(t) = 35e^{-10t}$ V for t > 0

$i(t) = 7e^{-10t}$ mA for t > 0

Circle the correct answer :

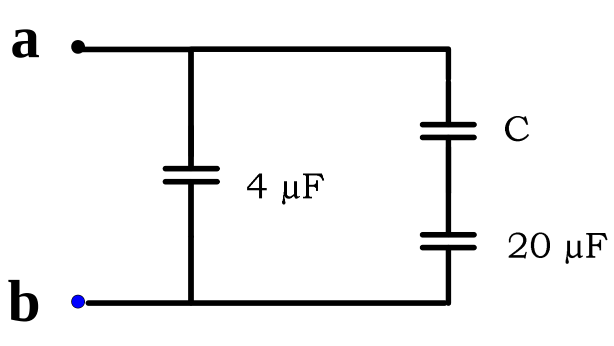
(a) **The value of the resistor R is**

(i) 7 kΩ (ii) 200 kΩ (iii) 35 kΩ (iv) 5 kΩ (v) $\frac{1}{5}$ kΩ (vi) $\frac{1}{200}$ kΩ

(a) **The value of the capacitor C is**

(i) 2000 µF (ii) 250 µF (iii) 200 µF (iv) 20 µF (v) 10 µF (vi) 5 µF

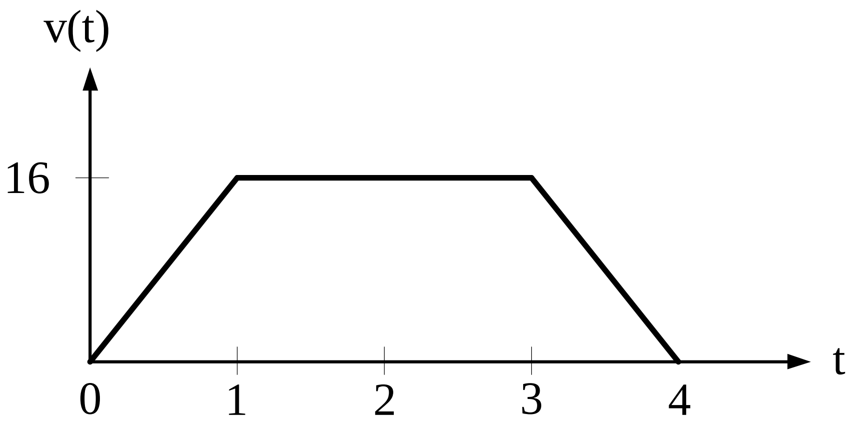
Part II The equivalent capacitance between terminals **a** and **b** in the circuit shown below is 8 µF.



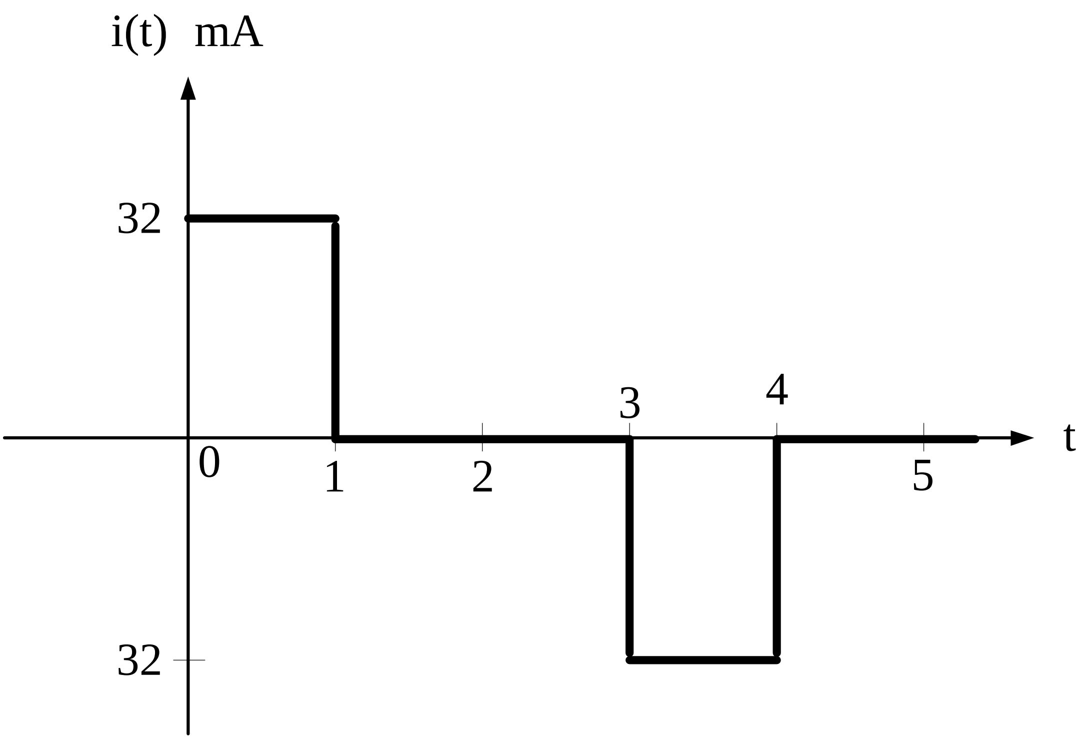
(c) The value of the capacitor C is (**Circle the correct answer**)

(i) 6 µF (ii) 5 µF (iii) 8 µF (iv) 2 µF (v) 10 µF (vi) 4 µF

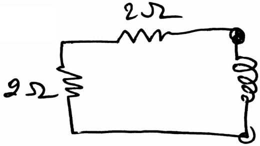
Part III The voltage v(t) across a 2 mF capacitor is shown below



(d) Plot current i(t) through the capacitor showing all **significant values** ?



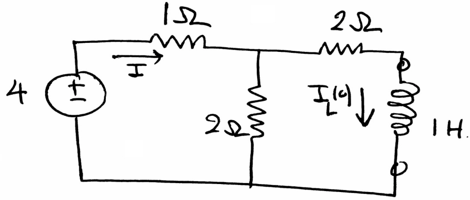
① After switching the circuit becomes



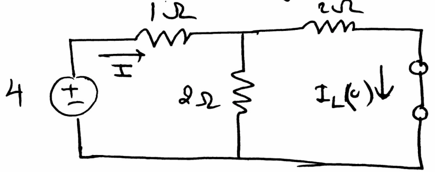
$R_{eq} = 2 + 2 = 4\,\Omega$

$\therefore \tau = \frac{L}{R} = \frac{1}{4}$ sec.

② Circuit before switching $t<0$ is



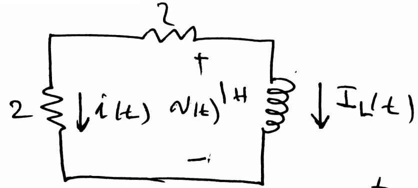
This circuit is equivalent to



$\therefore I = \frac{4}{1 + 2 /\!/ 2} = \frac{4}{1+1} = 2\,A$

Using current division

$I_L(0) = 1\ A = I_L(0^+)$



$$i(t) = -I_L(t) = -I_L(0^+)\, e^{-\frac{t}{\tau}}$$

$$= -e^{-4t} \qquad t > 0$$

③ $$v(t) = L\frac{dI_L(t)}{dt} = 1 \cdot \frac{d}{dt}\left(e^{-4t}\right)$$

$$= -4e^{-4t}$$

④ Energy in Inductor =

$$\frac{1}{2} L\, I_L(0)^2 = \frac{1}{2} \times 1 \times 1^2 = \frac{1}{2} \text{ Joule}$$