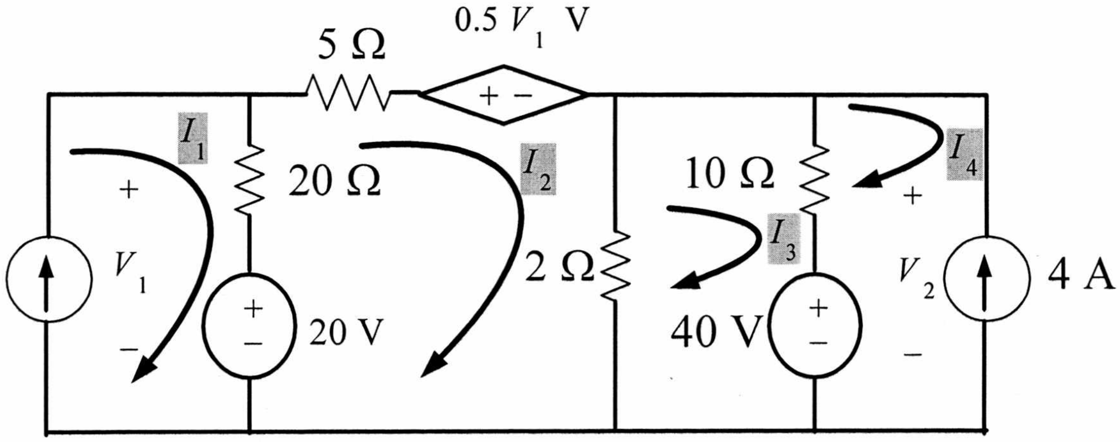
EE 202-Fall 2012(121)

HW3 **KEY**

Dr. Adil S. Balghonaim

Due 5/11/2012

Q1



By Inspection $I_1 = 2$ A $\quad I_4 = -4$ A

KVL on Mesh I_2 $\quad -20 + 20(I_2 - I_1) + 5I_2 + 0.5V_1 + 2(I_2 - I_3) = 0$

Since $V_1 = 20(I_1 - I_2) + 20 \quad I_1 = 2$ A $\Rightarrow 17I_2 - 2I_3 = 30$ -----(1)

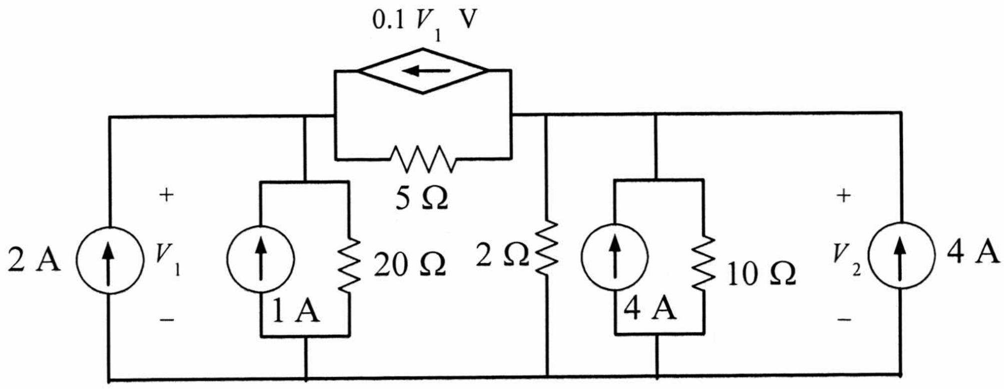
KVL on Mesh I_3 $\quad 2(I_3 - I_2) + 10(I_3 - I_4) + 40 = 0$

Since $\quad I_4 = -4$ A $\Rightarrow -2I_2 + 12I_3 = -80$ -----(2)

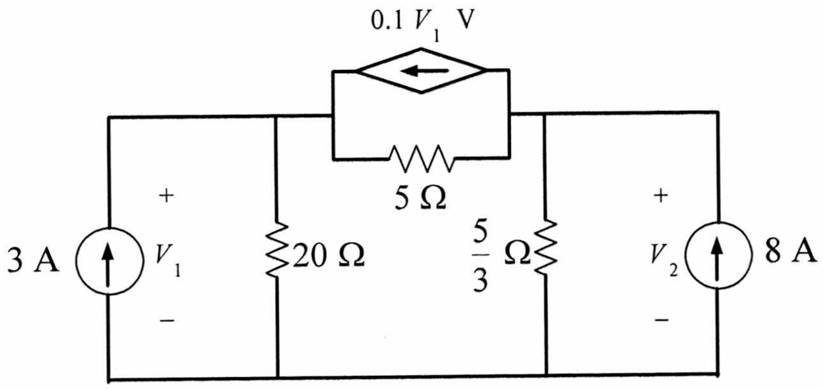
Solving (1) and (2) $\Rightarrow I_2 = 1$ A $\quad I_3 = -6.5$ A

$$P_{2\Omega} = 2(I_2 - I_3)^2 = 2(1+6.5)^2 = 112.5 \text{ W}$$

Q2 **Step 1: Source Transform the voltage sources**

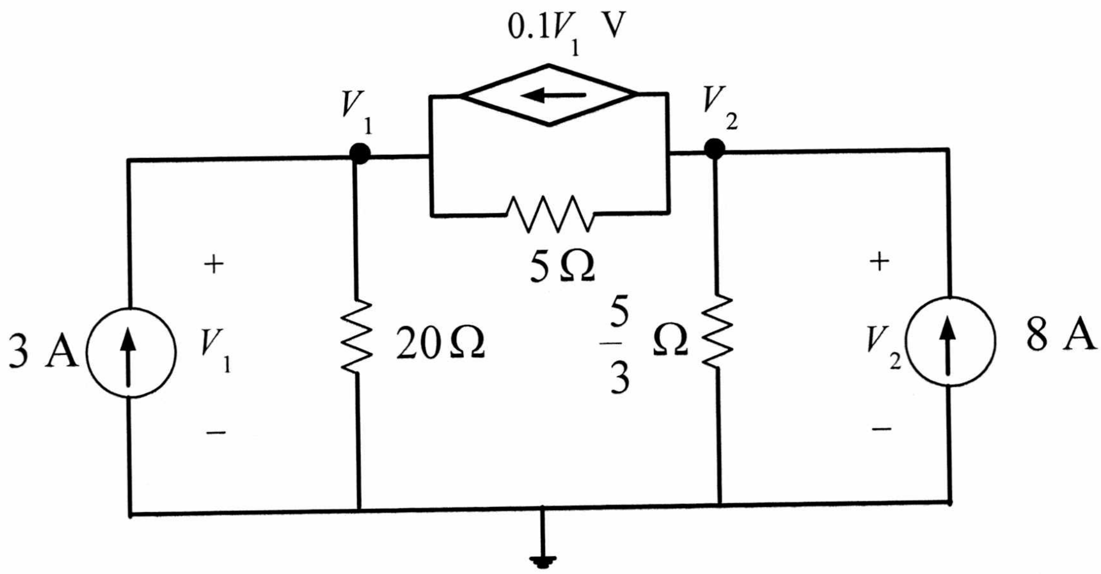


Step 2: Combine the currents sources and parallel resistors



Note : you can not transform the currents sources because that will destroy the required voltages namely V_1, V_2

Step 3 : Solve the remaining circuit using nodal analysis



KCl on the two nodes V_1, V_2 , we have

$$\frac{V_1}{20} + \frac{V_1 - V_2}{5} - 0.1V_1 - 3 = 0$$

$$\Rightarrow\ 0.15V_1 - 0.2V_2 = 3 \text{ --------(1)}$$

$$\frac{V_2}{5/3} + \frac{V_2 - V_1}{5} + 0.1V_1 - 8 = 0$$

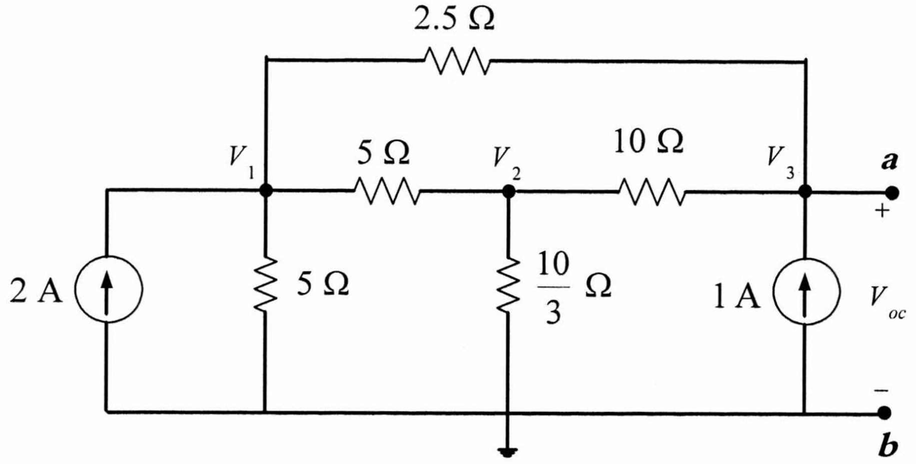
$$\Rightarrow\ -0.1V_1 + 0.8V_2 = 8 \text{ --------(2)}$$

$$\Rightarrow \begin{bmatrix} 0.15 & -0.2 \\ -0.1 & 0.8 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$$

Solving $\Rightarrow V_1 = 40$ V $\quad V_2 = 15$ V

Q3

(a) **Using Nodal Analysis were** $V_{oc} = V_3$



KCl on the three nodes V_1, V_2, V_3

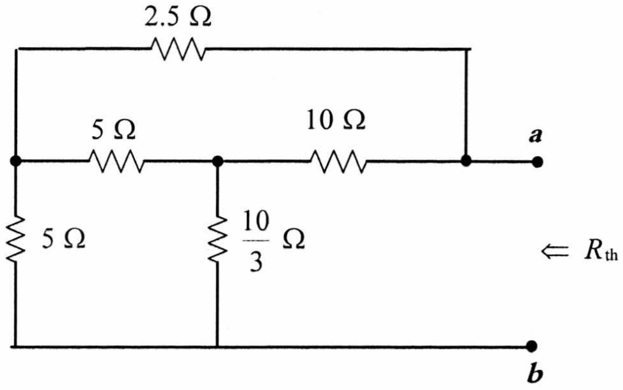
$$\frac{V_1}{5}+\frac{V_1-V_2}{5}+\frac{V_1-V_3}{2.5}-2=0 \Rightarrow \quad 0.8V_1-0.2V_2-0.4V_3=2 \text{ ----(1)}$$

$$\frac{V_2-V_1}{5}+\frac{V_2-V_3}{10}+\frac{V_2}{10/3}=0 \Rightarrow \quad -0.2V_1+0.6V_2-0.1V_3=0 \text{ ----(2)}$$

$$\frac{V_3-V_2}{10}+\frac{V_3-V_1}{2.5}-1=0 \quad \Rightarrow \quad -0.4V_1-0.1V_2+0.5V_3=1 \text{ ----(3)}$$

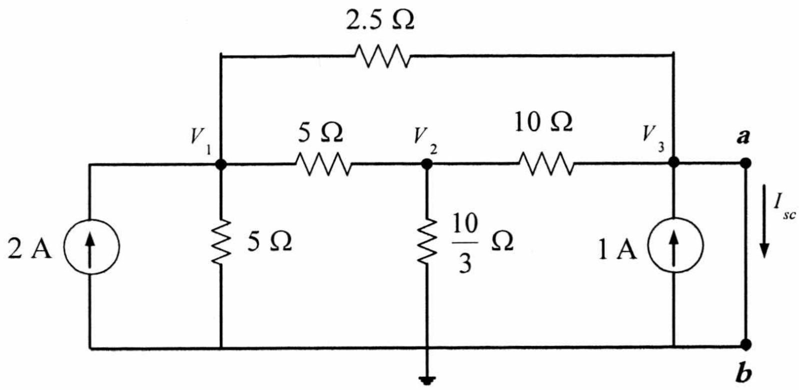
$$\begin{bmatrix} 0.8 & -0.2 & -0.4 \\ -0.2 & 0.6 & -0.1 \\ -0.4 & -0.1 & 0.5 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \quad \text{Solving } \Rightarrow V_{oc}=V_3=9.6\text{ V}$$

To find R_{th} , we deactivate all independent sources



Since the resistors can not be combined as series and parallel , we then find R_{th}

$R_{\text{th}} = \frac{V_{oc}}{I_{sc}}$. We already have V_{oc} , we then seek I_{sc} as follows :



Here $V_3 = 0$, KCl on the two nodes V_1, V_2

$$\frac{V_1}{5} + \frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{2.5} - 2 = 0 \Rightarrow \quad 0.8V_1 - 0.2V_2 = 2 \text{ ----(1)}$$

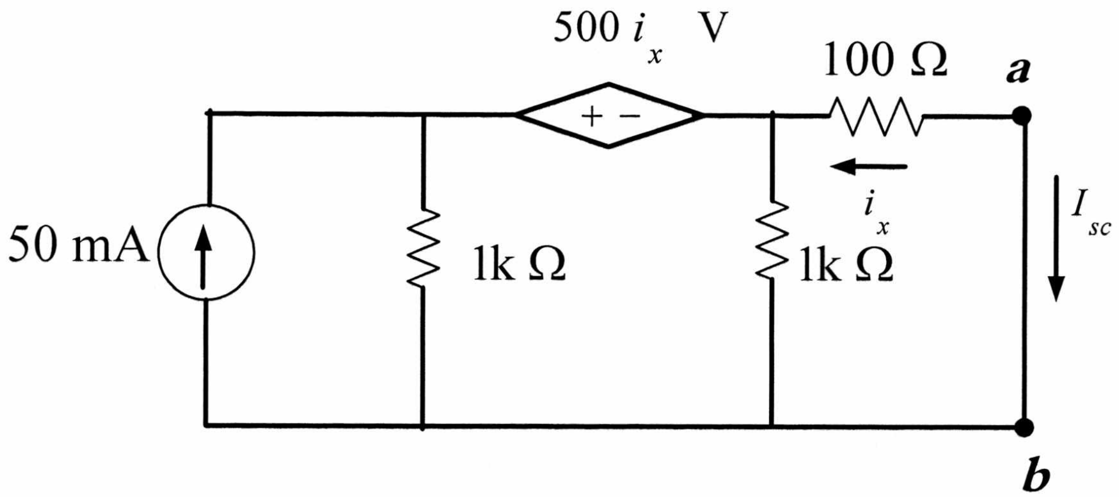
$$\frac{V_2 - V_1}{5} + \frac{V_2 - V_3}{10} + \frac{V_2}{10/3} = 0 \Rightarrow \quad -0.2V_1 + 0.6V_2 = 0 \text{ ----(2)}$$

$$\begin{bmatrix} 0.8 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad \text{Solving } \Rightarrow V_1 = 2.73 \text{ V} \quad V_2 = 0.91 \text{ V}$$

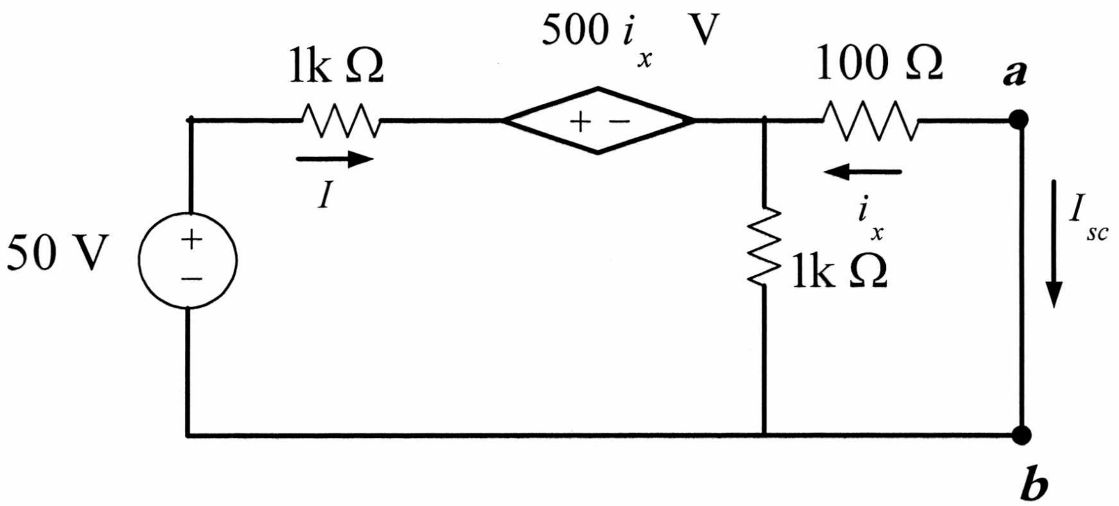
$$\frac{V_1 - V_3}{2.5} + \frac{V_2 - V_3}{10} + 1 = I_{sc} \quad \Rightarrow \frac{2.73}{2.5} + \frac{0.91}{10} + 1 = I_{sc} \Rightarrow I_{sc} = 2.183 \Rightarrow R_{\text{th}} = \frac{V_{oc}}{I_{sc}} = \frac{9.6}{2.183} = 4.4\ \Omega$$

(b) $R_L = R_{TH} = 4.4\ \Omega$ (c) $P_{\max} = \frac{V_{oc}^{\ 2}}{4R_{TH}} = \frac{9.6^2}{4(4.4)} = 5.24$ W

Q4



Source Transformation



KVL on the outer loop

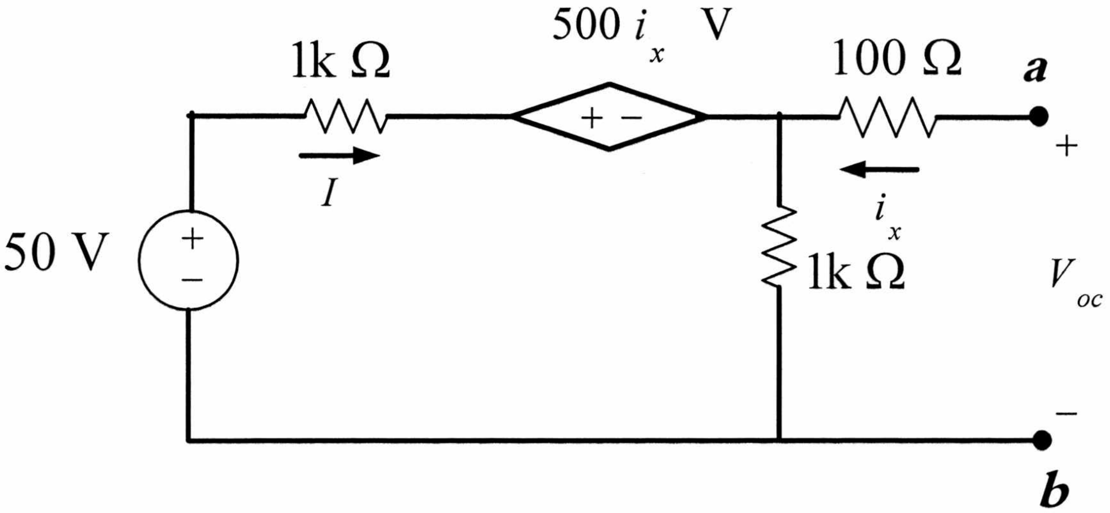
$$-50 + 1000I + 500i_x - 100i_x = 0 \;\Rightarrow\; 1000I + 400i_x = 50 \quad ---(1)$$

KVL on the inner loop

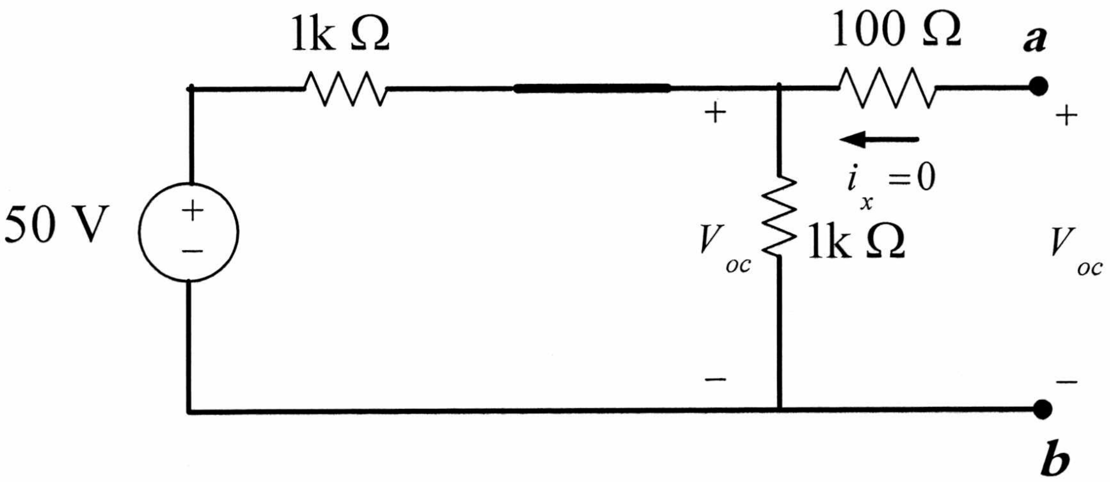
$$-1000(I + i_x) - 100i_x = 0 \;\Rightarrow\; 1000I + 1100i_x = 0 \quad ---(2)$$

Solving (1) and (2) for $i_x = -\frac{1}{14}$ A $\Rightarrow I_{sc} = -i_x = \frac{1}{14}$ A

Finding R_{th} : Since the circuit has a dependent source we cannot find R_{th} by combining resistors . We find R_{th} as $R_{\text{th}} = \frac{V_{oc}}{I_{sc}}$. Therefore we seek V_{oc} as follows :



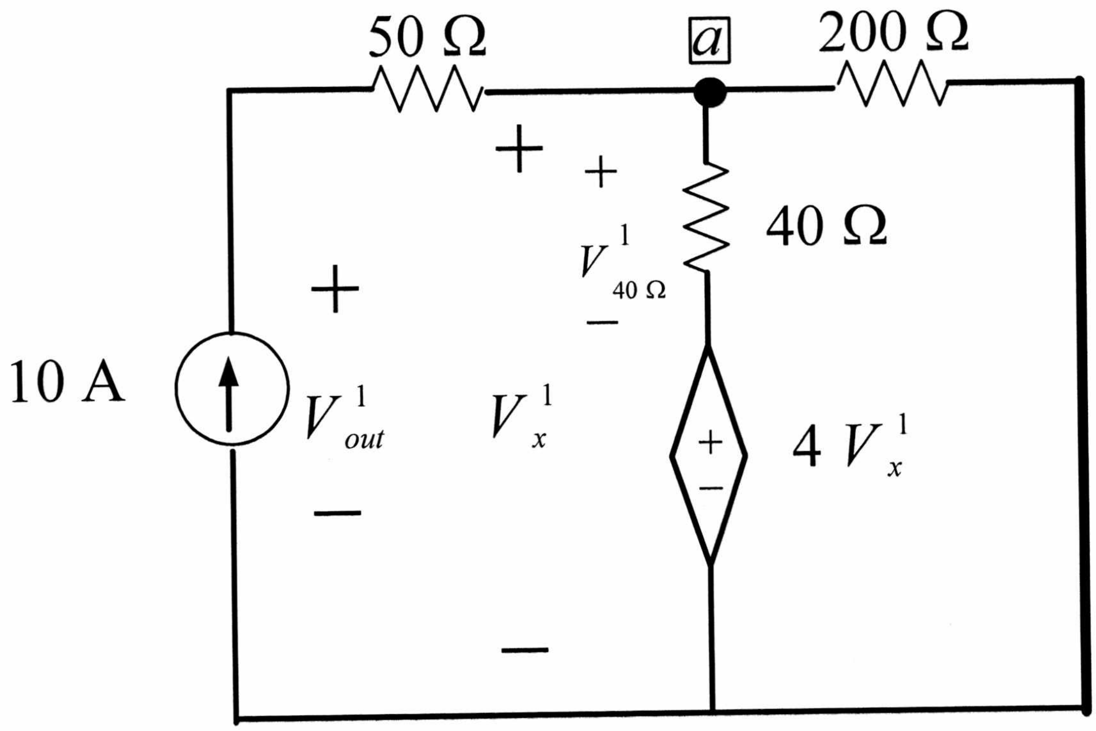
Here $i_x = 0$ which implies also that the dependent source $500i_x = 0$



$$V_{oc} = \frac{1k}{1k + 1k}(50) = 25 \text{ V}$$

$$\Rightarrow R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{25}{1/14} = 350\ \Omega$$

Q5 **Step 1 : Deactivate the independent voltage source**



Apply KCL at node a

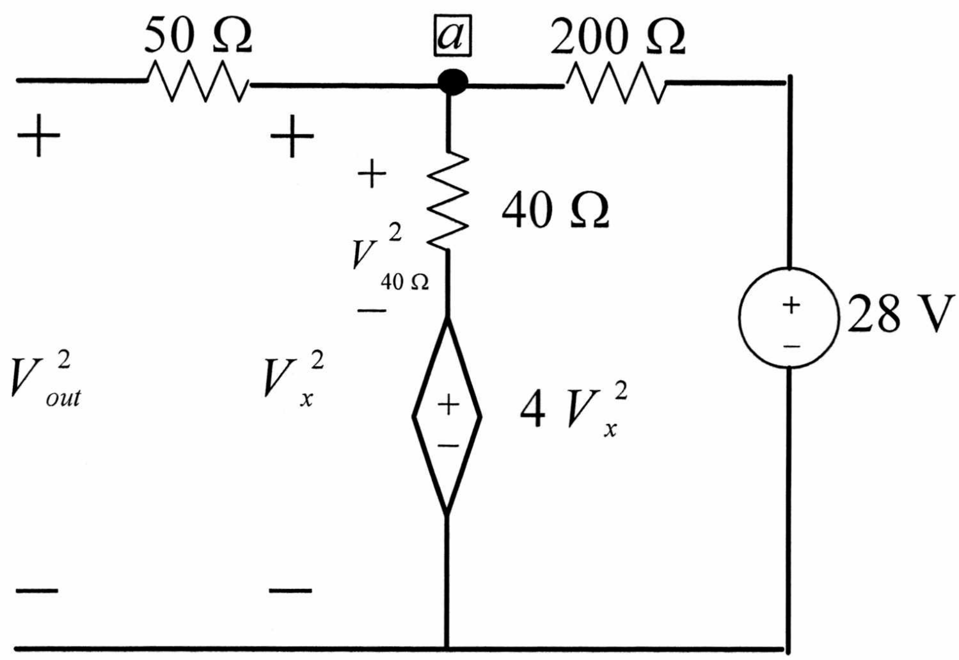
$$\frac{V_x^1}{200}+\frac{V_{40\,\Omega}^1}{40}-10=0 \quad \text{----(1)}$$

Since $V_{40\,\Omega}^1 = V_x^1 - 4V_x^1 = -3V_x^1$

$$(1) \Rightarrow \quad \frac{V_x^1}{200}+\frac{-3V_x^1}{40}-10=0 \Rightarrow V_x^1 = -\frac{1000}{7} = -142.857 \text{ V}$$

$$\text{KVL} \Rightarrow -V_{out}^1 + 50(10) + V_x^1 = 0 \Rightarrow V_{out}^1 = \frac{2500}{7} = 357.143 \text{ V}$$

Step 2 : Deactivate the independent current source



Apply KCL at node a

$$\frac{V_x^2 - 28}{200} + \frac{V^2_{40\,\Omega}}{40} = 0 \quad \text{----(2)}$$

Since $V^2_{40\,\Omega} = V_x^2 - 4V_x^2 = -3V_x^2$

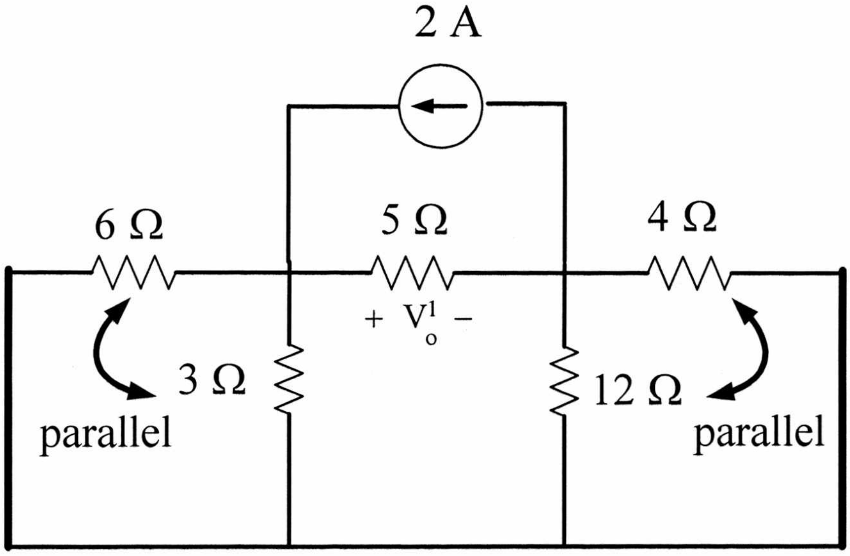
$$(2) \Rightarrow \quad \frac{V_x^2 - 28}{200} + \frac{-3V_x^2}{40} = 0 \;\Rightarrow\; V_x^2 = -2 \text{ V}$$

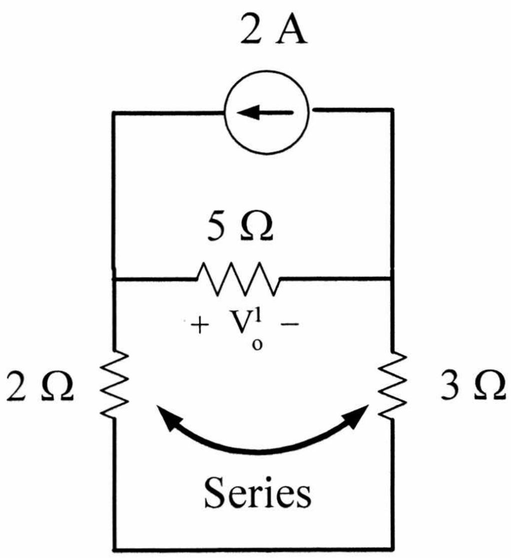
$V^2_{out} = V_x^2 = -2$ V

Step 3 : Combine the results in step 1 and step 2

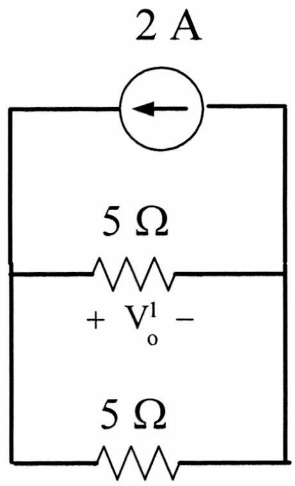
$$V_{out} = V^1_{out} + V^2_{out} = 357.143 + (-2) = 355.143 \text{ V}$$

Q6 **Step 1 : Deactivate the independent voltage sources**

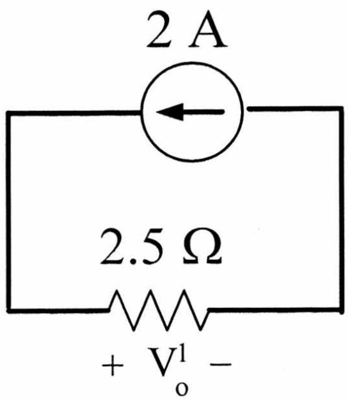




$\Rightarrow$

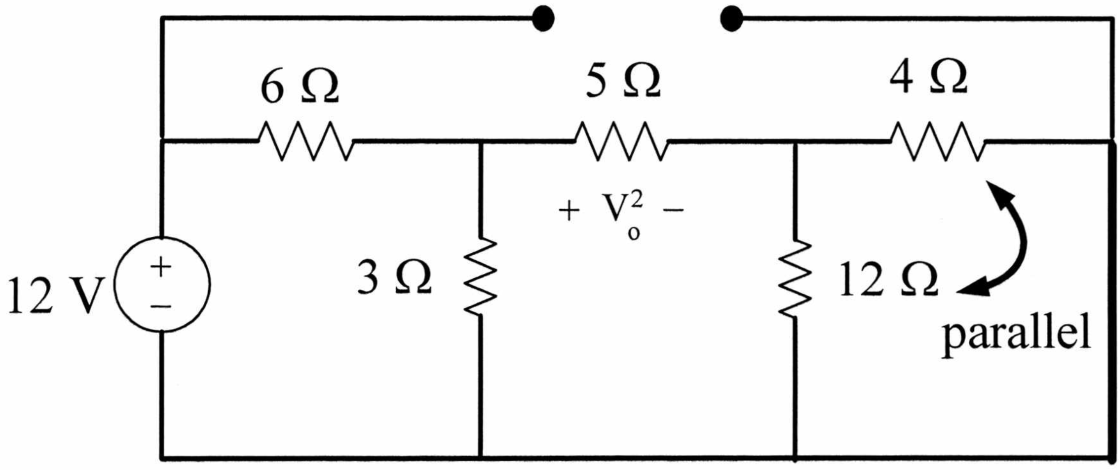


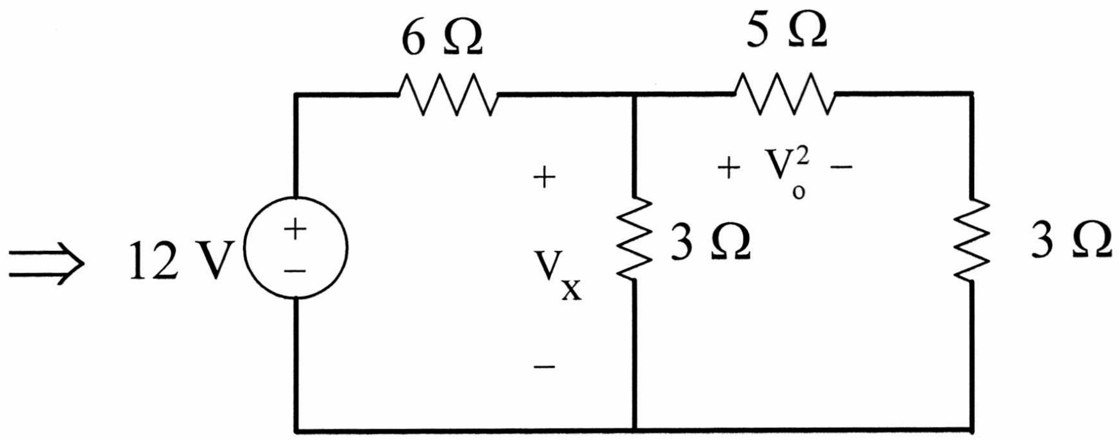
$\Rightarrow$



$$V_o^1 = (2.5)(2) = 5 \text{ V}$$

Step 2 : Deactivate the 19 V independent voltage source and the the 2 A V independent current source

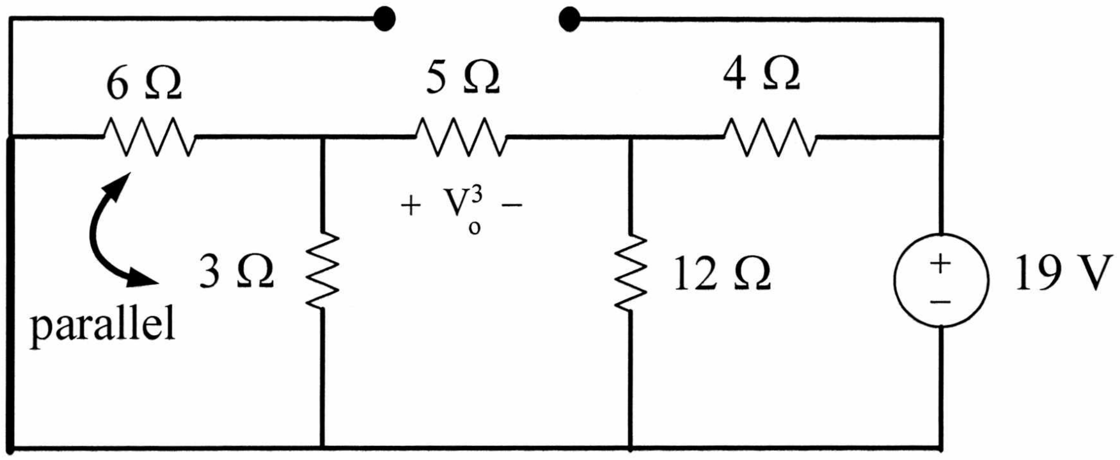


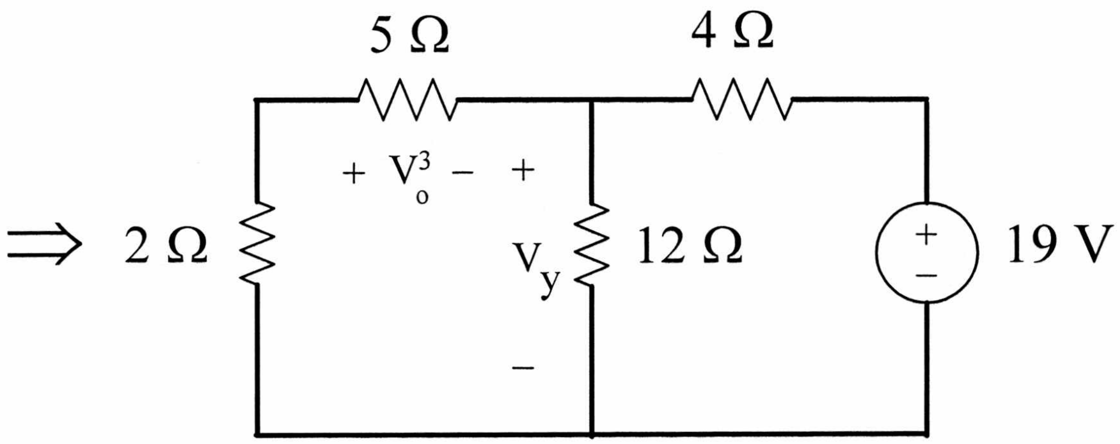


$$V_X = \frac{3\|(5+3)}{6+3\|(5+3)}12 = \frac{16}{5} = 3.2\text{ V}$$

$$V_o^2 = \frac{5}{5+3}V_X = \left(\frac{5}{8}\right)\left(\frac{16}{5}\right) = 2\text{ V}$$

Step 2 : Deactivate the 12 V independent voltage source and the the 2 A V independent current source





$$V_Y = \frac{12\|(5+2)}{4+12\|(5+2)}19 = \frac{399}{40} = 9.975 \text{ V}$$

$$V_o^3 = -\frac{5}{5+2}V_Y = -\left(\frac{5}{7}\right)\left(\frac{399}{40}\right) = -\frac{57}{8} = -7.125 \text{ V}$$

$$V_o = V_o^1 + V_o^2 + V_o^3 = 5+2+\ (-7.125\ ) = -0.125 \text{ V} = -125 \text{ mV}$$