EE 202-Fall 2012(121)

HW6 - Solution

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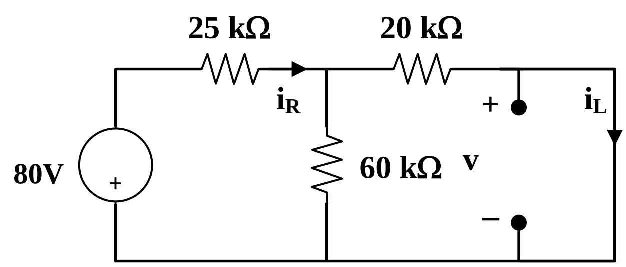
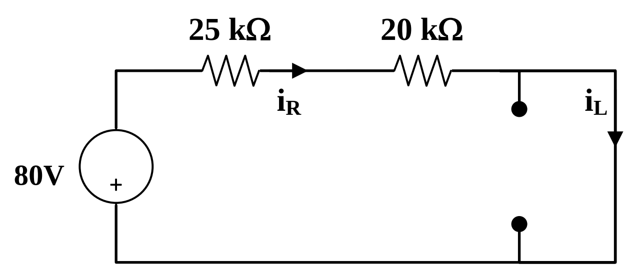
Due 15/12/2012

Q1

(a) At t = 0-, the equivalent circuit is shown in Figure (a).



(a)



(b)

60||20 = 15 kohms, $i_R(0-) = 80/(25 + 15) = 2mA$.

By the current division principle,

$$i_L(0-) = 60(2mA)/(60 + 20) = 1.5 \text{ mA}$$

$$v_C(0-) = 0$$

At t = 0+,

$$v_C(0+) = v_C(0-) = 0$$

$i_L(0+) = i_L(0-) =$ **1.5 mA**

$$80 = i_R(0+)(25 + 20) + v_C(0-)$$

$$i_R(0+) = 80/45k = \underline{\mathbf{1.778\ mA}}$$

But, $i_R = i_C + i_L$

$$1.778 = i_C(0+) + 1.5 \text{ or } i_C(0+) = \underline{\mathbf{0.278\ mA}}$$

(b) $v_L(0+) = v_C(0+) = 0$

But, $v_L = L di_L/dt$ and $di_L(0+)/dt = v_L(0+)/L = 0$

$$di_L(0+)/dt = \underline{\mathbf{0}}$$

Again, $80 = 45i_R + v_C$

$$0 = 45di_R/dt + dv_C/dt$$

But, $dv_C(0+)/dt = i_C(0+)/C = 0.278 \text{ mohms}/1\ \mu F = 278 \text{ V/s}$

Hence, $di_R(0+)/dt = (-1/45)dv_C(0+)/dt = -278/45$

$$di_R(0+)/dt = \underline{\mathbf{-6.1778\ A/s}}$$

Also, $i_R = i_C + i_L$

$$di_R(0+)/dt = di_C(0+)/dt + di_L(0+)/dt$$

$$-6.1788 = di_C(0+)/dt + 0, \text{ or } di_C(0+)/dt = \underline{\mathbf{-6.1788\ A/s}}$$

(c) As t approaches infinity, we have the equivalent circuit in Figure (b).

$$i_R(\infty) = i_L(\infty) = 80/45k = \underline{\mathbf{1.778\ mA}}$$

$$i_C(\infty) = Cdv(\infty)/dt = \underline{\mathbf{0}}.$$

Q2

In the circuit in Fig. 2, calculate $i_o(t)$ and $v_o(t)$ for t>0.

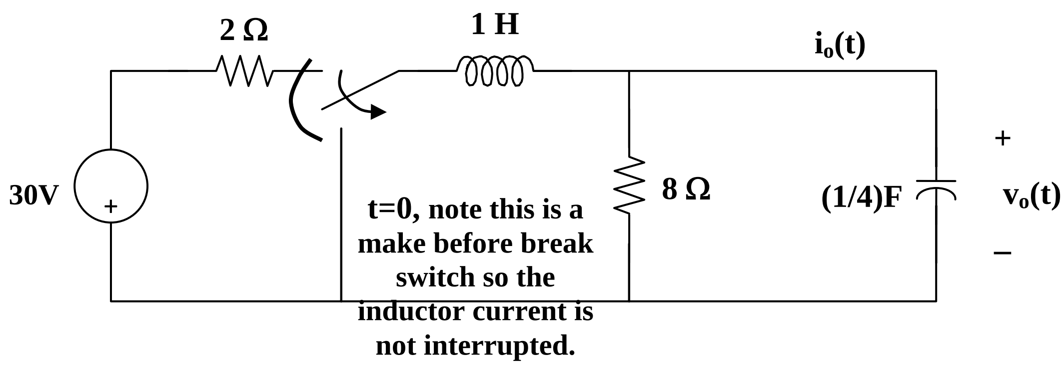


Figure 2 For Problem 2.

At $t = 0^-$, $v_o(0) = (8/(2 + 8)(30) = 24$

For t > 0, we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = 1/4$$
$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1 \text{x} 1/4} = 2$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{4 - (1/16)} = 1.9843$$

$$v_o(t) = (A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t}$$

$$v_o(0) = 30(8/(2+8)) = 24 = A_1 \text{ and } i_o(t) = C(dv_o/dt) = 0 \text{ when } t = 0.$$

$$dv_o/dt = -\alpha(A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t} + (-\omega_d A_1\sin\omega_d t + \omega_d A_2\cos\omega_d t)e^{-\alpha t}$$

$$\text{at } t = 0, \text{ we get } dv_o(0)/dt = 0 = -\alpha A_1 + \omega_d A_2$$

Thus, $A_2 = (\alpha/\omega_d)A_1 = (1/4)(24)/1.9843 = 3.024$

$v_o(t) =$ **$(24\cos 1.9843t + 3.024\sin 1.9843t)e^{-t/4}$ volts**.

$i_0(t) = Cdv/dt = 0.25[-24(1.9843)\sin 1.9843t + 3.024(1.9843)\cos 1.9843t - 0.25(24\cos 1.9843t) - 0.25(3.024\sin 1.9843t)]e^{-t/4}$

$=$ **$[0.000131\cos 1.9843t - 12.095\sin 1.9843t]e^{-t/4}$ A**.

Q3

Given that $s_1 = -10$ and $s_2 = -20$, we recall that

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -10, -20$$

Clearly, $s_1 + s_2 = -2\alpha = -30$ or $\alpha = 15 = R/(2L)$ or $R = 60L$ (1)

$$s_1 = -15 + \sqrt{15^2 - \omega_o^2} = -10 \text{ which leads to } 15^2 - \omega_o^2 = 25$$

$$\text{or } \omega_o = \sqrt{225 - 25} = \sqrt{200} = 1/\sqrt{LC}, \text{ thus } LC = 1/200 \quad (2)$$

Since we have a series RLC circuit, $i_L = i_C = Cdv_C/dt$ which gives,

$$i_L/C = dv_C/dt = [200e^{-20t} - 300e^{-30t}] \text{ or } i_L = 100C[2e^{-20t} - 3e^{-30t}]$$

But, i is also = $20\{[2e^{-20t} - 3e^{-30t}]x10^{-3}\}$ = $100C[2e^{-20t} - 3e^{-30t}]$

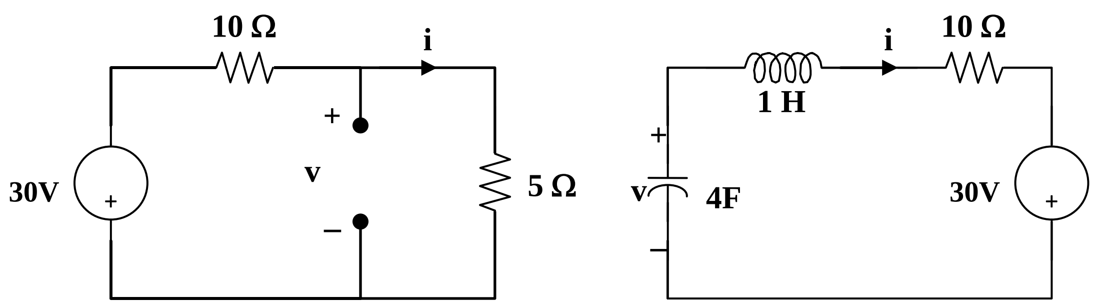
Therefore, C = $(0.02/10^2)$ = **200 μF**

L = 1/(200C) = **25 H**

R = 30L = **750 ohms**

Q4

We may transform the current sources to voltage sources. For t = 0^-, the equivalent circuit is shown in Figure (a).



(a) (b)

i(0) = 30/15 = 2 A, v(0) = 5x30/15 = 10 V

For t > 0, we have a series RLC circuit.

$$\alpha = R/(2L) = 5/2 = 2.5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4} = 0.25, \text{ clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2.5 \pm \sqrt{6.25 - 0.25} = -4.95, -0.05$$

$$v(t) = V_s + [A_1e^{-4.95t} + A_2e^{-0.05t}],\ v = 20.$$

$$v(0) = 10 = 20 + A_1 + A_2 \qquad (1)$$

$$i(0) = Cdv(0)/dt \text{ or } dv(0)/dt = 2/4 = 1/2$$

Hence,

$$½ = -4.95A_1 - 0.05A_2 \qquad (2)$$

From (1) and (2), $A_1 = 0$, $A_2 = -10$.

$$v(t) = \underline{\mathbf{\{20 - 10e^{-0.05t}\}\ V}}$$

Q5

For $t = 0-$, we obtain $i(0) = -6/(1 + 2) = -2$ and $v(0) = 2x1 = 2$.

For $t > 0$, the voltage is short-circuited and we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = (1)/(2x1x0.25) = 2$$
$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.25} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus, $i(t) = [(A + Bt)e^{-2t}]$, $i(0) = -2 = A$

$$v = Ldi/dt = [Be^{-2t}] + [-2(-2 + Bt)e^{-2t}]$$

$$v_o(0) = 2 = B + 4 \text{ or } B = -2$$

Thus, $i(t) = \underline{\mathbf{[(-2 - 2t)e^{-2t}]\ A}}$

and $v(t) = \underline{\mathbf{[(2 + 4t)e^{-2t}]\ V}}$