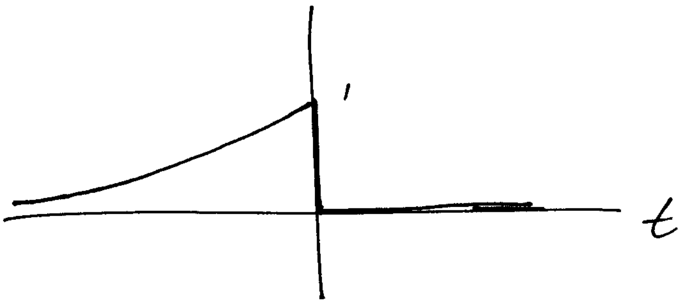
5.1 b) $e^{at}u(-t)$ $a>0$



set it to 0.

$$\mathcal{F}\{e^{at}u(-t)\} = \int_{-\infty}^{\infty} e^{+at}\cdot u(-t)\cdot e^{-j\omega t}\,dt$$

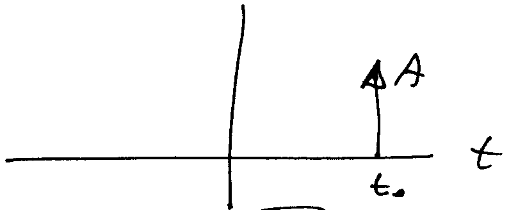
$$= \int_{-\infty}^{0} e^{+at}\cdot e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{0} e^{(+a-j\omega)t}\,dt$$

$$= \frac{1}{-a-j\omega}\cdot e^{(+a-j\omega)t}\Big|_{-\infty}^{0}$$

$$= \frac{1}{-a-j\omega} - 0 = \boxed{-\frac{1}{a+j\omega}}$$

c) $A\,\delta(t-t_0)$



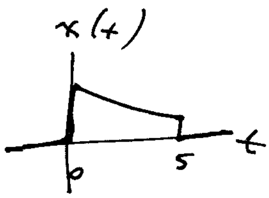
$$\mathcal{F}\{A\,\delta(t-t_0)\} = \int_{-\infty}^{\infty} A\,\delta(t-t_0)\cdot e^{-j\omega t}\,dt$$

$$= A\int_{-\infty}^{\infty} \delta(t-t_0)\cdot \underbrace{e^{-j\omega t_0}}_{\text{constant}}\,dt$$

$$= A\cdot e^{-j\omega t_0}\underbrace{\int_{-\infty}^{\infty}\delta(t-t_0)\,dt}_{=1}$$

$$\boxed{= A\cdot e^{-j\omega t_0}}$$

5.2 b) $x(t) = e^{-2t}\left[u(t) - u(t-5)\right]$



$$X(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt$$

$$= \int_0^5 e^{-2t}\cdot e^{-j\omega t}\,dt = \int_0^5 e^{(-2-j\omega)t}\,dt$$

$$= \frac{1}{-2-j\omega}\cdot e^{(-2-j\omega)t}\Big|_0^5 = \boxed{-\frac{1}{2+j\omega}\left[e^{5(-2-j\omega)} - 1\right]}$$

External (a) Find the Fourier transform of

$$x(t) = \frac{d}{dt}\left[\frac{3}{9+(t+2)^2}\right]$$

Duality

$$\frac{3}{(3)^2+t^2} \rightleftharpoons \frac{\cancel{2}\pi}{\cancel{2}} \cdot e^{-3|w|} = \pi e^{-3|w|}$$

Time Shift

$$\frac{3}{(3)^2+(t+2)^2} \rightleftharpoons \pi \cdot e^{-3|w|} \cdot e^{j2w}$$

Differentiation

$$\frac{d}{dt}\left[\frac{3}{(3)^2+(t+2)^2}\right] \rightleftharpoons \boxed{jw\pi \cdot e^{-3|w|} \cdot e^{j2w}}$$

(b) Find the Inverse F.T of

$$Y(w) = 9 \operatorname{sinc}^2(4[w-3])$$

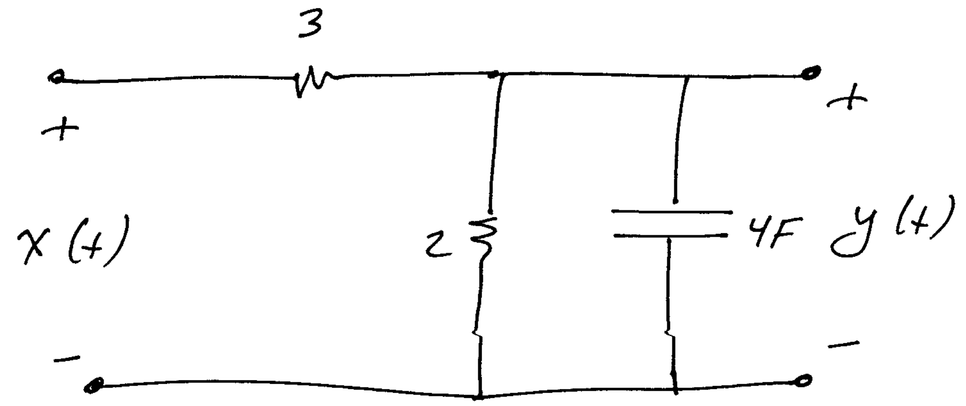
$$\frac{9}{8} \operatorname{tri}\left(\frac{t}{8}\right) \rightleftharpoons 9 \operatorname{sinc}^2(4w)$$

"T = 8" see table 5.2 line 16

$$\frac{9}{8} \operatorname{tri}\left(\frac{t}{8}\right) \cdot e^{j3t} \rightleftharpoons 9 \operatorname{sinc}^2(4[w-3])$$

External

The circuit shown below has input $x(t)$ and output $y(t)$



(a) Find the transfer function of the circuit

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2 \,//\, \frac{1}{j4\omega}}{2 \,//\, \frac{1}{j4\omega} + 3} = \frac{\frac{\frac{2}{j4\omega}}{2+\frac{1}{j4\omega}}}{\frac{\frac{2}{j4\omega}}{2+\frac{1}{j4\omega}} + 3}$$

$$= \frac{\frac{2}{j4\omega}}{\frac{2}{j4\omega} + 3\left(2+\frac{1}{j4\omega}\right)}$$

$$= \frac{2}{2 + 3\left(2+\frac{1}{j4\omega}\right)\cdot j4\omega}$$

$$= \frac{2}{2+1+6\cdot j4\omega} = \frac{2}{3+j24\omega}$$

(b) Find impulse response of the circuit.

$$h(t) = \mathcal{F}^{-1}\{H(\omega)\} = \mathcal{F}^{-1}\left\{\frac{2}{3+j24\omega}\right\}$$

$$\frac{2}{3+j24\omega} = \frac{2/24}{\frac{3}{24}+j\omega}$$

This is similar to Line 18 of table 5.2 on page 223.

$$e^{-at}u(t) \rightleftharpoons \frac{1}{a+j\omega}$$

$$h(t) = \frac{2}{24}\cdot e^{-\frac{3}{24}}u(t)$$

$$\boxed{h(t) = \frac{1}{12}\cdot e^{-\frac{1}{8}t}u(t)}$$

(c) Let the input to circuit be $x(t) = \delta(t-2)$

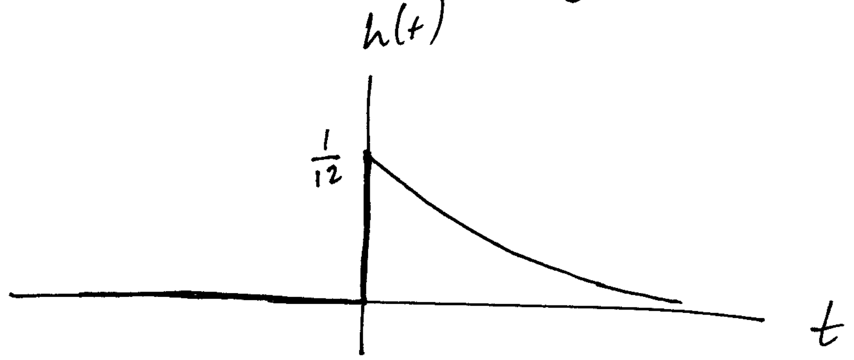
Find $y(t)$

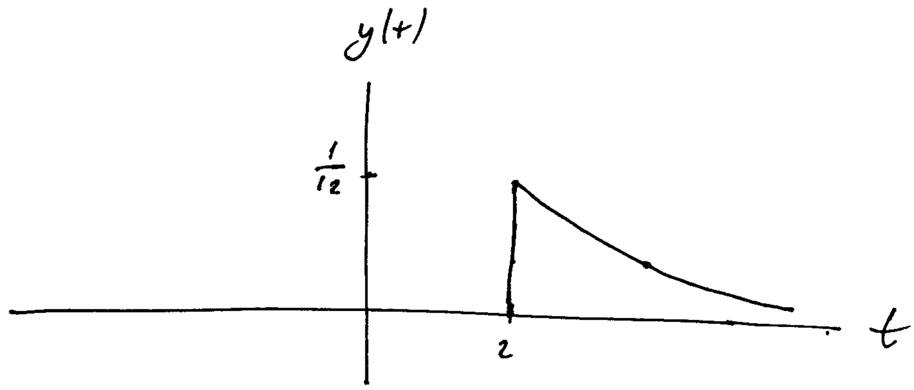
$$\left.\begin{aligned} X(\omega) &= e^{-j2\omega} \\ H(\omega) &= \frac{2}{3+j24\omega} \end{aligned}\right\} \quad Y(\omega) = \frac{2\cdot e^{-j2\omega}}{3+j24\omega}$$

$$X(\omega) = \frac{1}{12} \frac{e^{-j2\omega}}{\frac{1}{8} + j\omega}$$

$$y(t) = \frac{1}{12} \cdot e^{-\frac{1}{8}(t-2)} \cdot u(t-2)$$

(d) Sketch $h(t)$ and $y(t)$





External

A system is found to have a transfer function $H(\omega)$ given by

$$H(\omega) = \frac{3j\omega + 2}{4(j\omega)^2 + 2j\omega - 5}$$

Find the time domain differential Equation that describes the above system

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{3j\omega + 2}{4(j\omega)^2 + 2j\omega - 5}$$

Rewrite the above relation as

$$\left[4(j\omega)^2 + 2j\omega - 5\right] Y(\omega) = \left[3j\omega + 2\right] X(\omega)$$

Expand

$$4(j\omega)^2 Y(\omega) + 2j\omega Y(\omega) - 5Y(\omega) = 3j\omega X(\omega) + 2X(\omega)$$

Take the inverse F.T. of each term

Remember that

$$\frac{d^n}{dt^n} z(t) \rightleftharpoons (j\omega)^n Z(\omega)$$

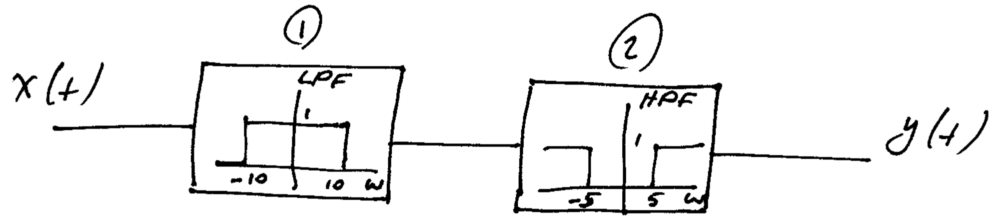
So,

$$\boxed{4\frac{d^2}{dt}y(t) + 2\frac{d}{dt}y(t) - 5y(t) = 3\frac{d}{dt}x(t) + 2x(t)}$$

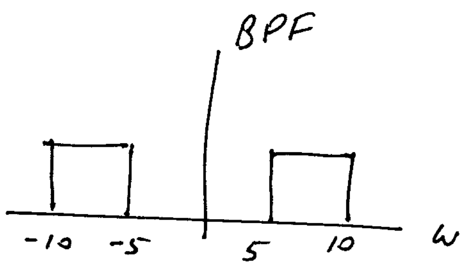
Starting from this differential equation and taking the F.T. of each term then collecting coefficents of $Y(\omega)$ togather and coefficients of $X(\omega)$ togather and writing $H(\omega) = \frac{Y(\omega)}{X(\omega)}$ gives the transfer function

External For the filters connected as shown below, determine the type of filter that results in each case

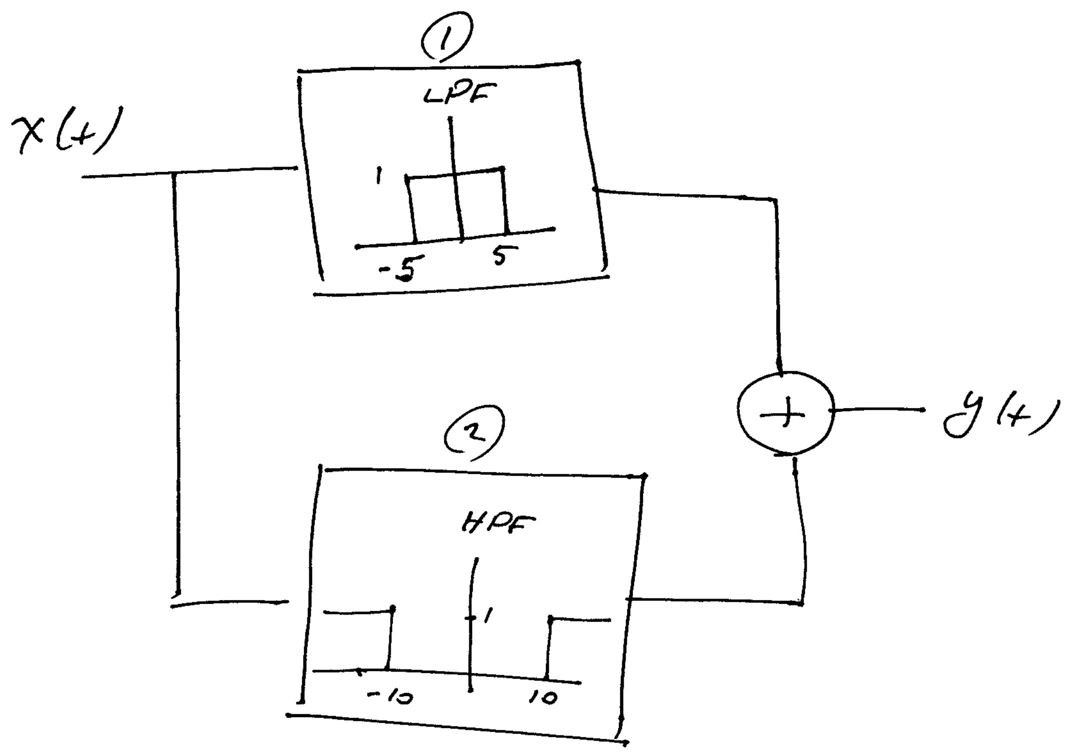
(a)



The LPF removes all frequency components above 10 rad/s. The filtered signal is filtered again where freq below 5 rad/s are removed. So, effectively, these two filters act as a single Band Pass filter

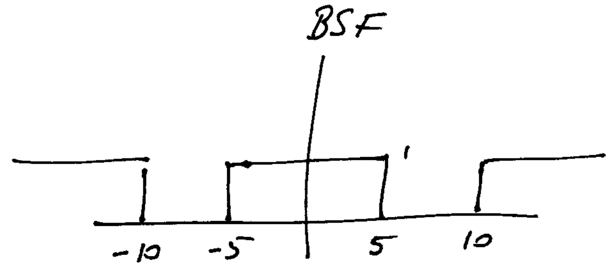


(b)



$y(t)$ is the combination of outputs of both filters. The LPF passes freq from 0 to 5 while the HPF passes freq from 10 to ∞. The result is that all freq from 0 to ∞ except freq. from 5 to 10 will pass

$\Longrightarrow$ Band stop filter



External

Find the Fourier Transform of

(a) $x(t) = \text{sinc}(4t) + \text{rect}\left(\frac{t-3}{6}\right)$

$\text{sinc}(4t) \rightleftharpoons \frac{\pi}{4}\,\text{rect}\left(\frac{\omega}{8}\right)$

$\text{rect}\left(\frac{t}{6}\right) \rightleftharpoons 6 \cdot \text{sinc}(3\omega)$

$\text{rect}\left(\frac{t-3}{6}\right) \rightleftharpoons 6\,\text{sinc}(3\omega) \cdot e^{-j3\omega}$

So, $X(\omega) = \frac{\pi}{4}\,\text{rect}\left(\frac{\omega}{8}\right) + 6\,\text{sinc}(3\omega) \cdot e^{-j3\omega}$

(b) $y(t) = 2\,\text{sinc}\left(\frac{t}{3}\right) \cdot \delta(t)$

Method 1

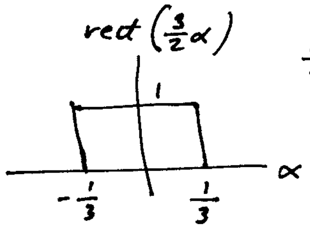
$\text{sinc}\left(\frac{t}{3}\right) \rightleftharpoons 3\pi \cdot \text{rect}\left(\frac{3}{2}\omega\right)$

$2\delta(t) \rightleftharpoons 2$

Multiplication in time gives convolution in freq. multiplied by $\frac{1}{2\pi}$

$2\,\text{sinc}\left(\frac{t}{3}\right) \cdot \delta(t) \rightleftharpoons \frac{1}{2\pi}\left[2 * 3\pi\,\text{rect}\left(\frac{3}{2}\omega\right)\right]$

$$2 * 3\pi \ \mathrm{rect}\left(\frac{3}{2}w\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2 \cdot 3\pi \ \mathrm{rect}\left(\frac{3}{2}\alpha\right) d\alpha$$



$$\frac{3}{2}\alpha = \frac{\alpha}{\frac{2}{3}}$$

$$= \frac{6\pi}{2\pi}\int_{-\infty}^{\infty} \mathrm{rect}\left(\frac{\alpha}{\frac{2}{3}}\right) d\alpha$$

$$= \frac{\cancel{6}\cancel{\pi}}{\cancel{2}\cancel{\pi}} \cdot \frac{\cancel{2}}{\cancel{3}} = 2$$

So, $Y(w) = 2$

Method 2

We observe that $y(t) = 2 \, \mathrm{sinc}\left(\frac{t}{3}\right) \cdot \delta(t)$

$$= 2 \, \mathrm{sinc}(0) \cdot \delta(t)$$

$$= 2 \, \delta(t)$$

So, $Y(w) = 2$

(c) $$z(t) = \frac{\cos(10\pi t)}{3+jt}$$

Since $$e^{-3t}u(t) \rightleftharpoons \frac{1}{3+j\omega}$$

So $$\frac{1}{3+jt} \rightleftharpoons 2\pi e^{-3(-\omega)} u(-\omega)$$

$$= 2\pi e^{3\omega} u(-\omega)$$

$$\frac{\cos(10\pi t)}{3+jt} \rightleftharpoons \pi \Big[e^{3(\omega-10\pi)} \cdot u(-(\omega-10\pi)) + e^{3(\omega+10\pi)} \cdot u(-(\omega+10\pi)) \Big]$$

External

The signal $x(t)$ is given by $x(t) = \dfrac{6}{(3)^2 + t^2}$.

Find

(a) Total Energy of $x(t)$ in time domain

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} \frac{36}{(9+t^2)^2} \, dt$$

$$= \underbrace{\cancel{\left. \frac{2t}{(9+t^2)} \right|_{-\infty}^{\infty}}}_{=0 \text{ at } \pm\infty} + \frac{2}{3} \cdot \tan^{-1}\left(\frac{t}{3}\right) \Big|_{-\infty}^{\infty}$$

I used Matlab to evaluate the integral

$$= \frac{2}{3}\left[\tan^{-1}(\infty) - \tan^{-1}(-\infty)\right] = \frac{2}{3}\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = 2.0944$$

(b) Total Energy of $x(t)$ in freq domain
(Using Parseval's Theorem)

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$$

$$X(\omega) = 2\pi e^{-3|-\omega|} = 2\pi e^{-3|\omega|}$$

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} 4\pi^2 \cdot \left[e^{-3|\omega|}\right]\cdot\left[e^{-3|\omega|}\right] d\omega$$

$$= \frac{1}{\cancel{2\pi}}\int_{-\infty}^{\infty} \cancel{4}^{2}\pi^{\cancel{2}} e^{-6|\omega|} \, d\omega$$

$$= 2\pi\left[\int_{-\infty}^{0} e^{+6\omega} \, d\omega + \int_{0}^{\infty} e^{-6\omega} \, d\omega\right]$$

Same

$$= 4\pi \cdot \int_{0}^{\infty} e^{-6\omega} \, d\omega = \frac{4\pi}{-6} \cdot e^{-6\omega}\Big|_{0}^{\infty}$$

$$= 0 - \frac{4\pi}{-6}(-1) = \frac{4\pi}{6} = 2.0944$$

Same as the previous value