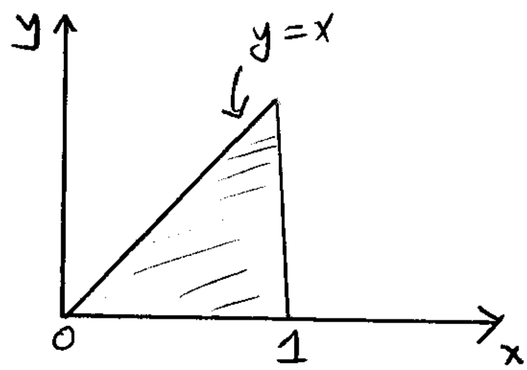
EE 315 – Fall 2011(111)
Quiz 4

SER	ID	NAME KEY

If X and Y are random variables with a joint density function

$$f_{XY}(x,y)=\begin{cases} 2 & 0<y\le x<1 \\ 0 & \text{else} \end{cases}$$



(a) Are X and Y independent, *explain*?

(b) Find $f_{X|Y}(x\,|\,y)$?

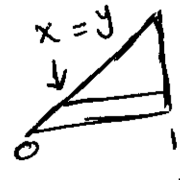
(c) $P\left(0<x<\frac{1}{3}\,\middle|\,y=\frac{1}{8}\right)$?

solution

(a) $f_{XY}(x,y) \overset{?}{=} f_X(x)\, f_Y(y)$

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy = \int_{0}^{y=x} 2\,dy = 2x \qquad 0<x<1$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx = \int_{x=y}^{1} 2\,dx = 2(1-y) \qquad 0<y<1$$



Since $f_{XY}(x,y) \neq f_X(x)\, f_Y(y) \Rightarrow$ not independent.

(b) $f_{X|Y}(x|y) = \dfrac{f_{XY}(x,y)}{f_Y(y)} = \dfrac{2}{2(1-y)} = \dfrac{1}{1-y} \qquad y\le x<1, \; 0<y<1$

(c) $P\left(0 < X < \frac{1}{3} \middle| y = \frac{1}{8}\right) = \int_0^{1/3} f_{X|Y}\left(x \middle| y = \frac{1}{8}\right) dx$

$$f_{X|Y}\left(x \middle| y = \tfrac{1}{8}\right) = \frac{1}{1-\frac{1}{8}} \qquad \frac{1}{8} \le X < 1$$

$$= \frac{8}{7} \qquad \frac{1}{8} < X < 1$$

$$= 0 \qquad \text{else}$$

$$\Rightarrow P\left(0 < X < \tfrac{1}{3} \middle| y = \tfrac{1}{8}\right) = \int_0^{1/8} 0 \, dx + \int_{1/8}^{1/3} \frac{8}{7} dx$$

$$= \frac{8}{7} x \Big|_{1/8}^{1/3}$$

$$= \frac{8}{7}\left(\frac{1}{3} - \frac{1}{8}\right)$$

$$= \frac{5}{21}$$