

EE 315 – Fall 2011(111)
Quiz 3

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If X is a random variable with density function

$$f_X(x) = \begin{cases} 6x(1-x) & 0 < x < 1 \\ 0 & \text{else} \end{cases}$$

If $Y = X^3$, find $f_Y(y)$?

Solution

method (1)

using $f_Y(y) = \frac{f_X(x)|_{x=T^{-1}(y)}}{|dy/dx|_{x=T^{-1}(y)}}$

$$x = y^{1/3} \quad \left| \frac{dy}{dx} \right| = 3x^2 = 3(y^{1/3})^2 = 3y^{2/3}$$

~~f_X~~ $f_Y(y) = \begin{cases} \frac{6(y^{1/3})(1-y^{1/3})}{3y^{2/3}} & 0 < y < 1 \\ 0 & \text{else} \end{cases}$

$$= \begin{cases} 2y^{-1/3} - 2 & 0 < y < 1 \\ 0 & \text{else} \end{cases}$$

\Rightarrow

method 2

$$f_Y(y) = \frac{d}{dy} F_Y(y)$$

$$\begin{aligned} F_Y(y) &= P(Y \leq y) = P(X \leq y^{1/3}) \\ &= \int_0^{y^{1/3}} 6x(1-x) dx \\ &= 3y^{2/3} - 2y \end{aligned}$$

$$f_Y(y) = \frac{d}{dy} F_Y(y) = 2y^{-1/3} - 2 \quad 0 < y < 1$$