| SER	ID	NAME KEY

If X is a random variable with uniform density function between 0 and 1

(a) Plot the cumulative distribution $F_X(x)$?

(b) Find $P\left(\dfrac{1}{3} < X < \dfrac{1}{2}\right)$?

(c) Derive and plot the conditional density function $f_X\left(x \mid X \le 0.5\right)$?

Solution

(a) $f_X(x)$
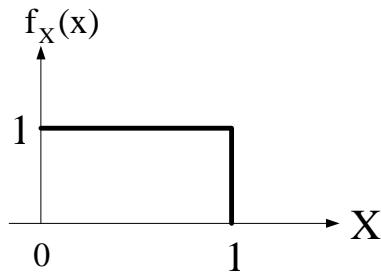
$$F_X(x) = \int_{-\infty}^{x} f_X(x')dx' = \begin{cases} 0 & X \le 0 \\ \int_0^x 1\,dx' = x & 0 \le X \le 1 \\ 1 & 0 \le X \le 1 \end{cases}$$

$F_X(x)$
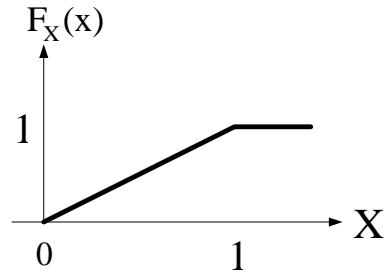
(b) $P\left(\dfrac{1}{3} < X < \dfrac{1}{2}\right) = \int_{\frac{1}{3}}^{\frac{1}{2}} (1)dx = \dfrac{1}{6}$

$$f_X\left(x\mid X\le 0.5\right)=\frac{d}{dx}F_X\left(x\mid X\le 0.5\right)$$

$$F_X\left(x\mid X\le 0.5\right)=P\left(X\le x\mid X\le 0.5\right)=\frac{P\left(X\le x\cap X\le 0.5\right)}{P\left(X\le 0.5\right)}$$

$$=\begin{cases}\dfrac{P\left(X\le x\right)}{P\left(X\le 0.5\right)} & x<0.5\\[2ex]\dfrac{P\left(X\le 0.5\right)}{P\left(X\le 0.5\right)}=1 & x\ge 0.5\end{cases}=\begin{cases}\dfrac{F_X(x)}{\underbrace{F_X(0.5)}_{\frac{1}{2}}} & x<0.5\\[3ex]1 & x\ge 0.5\end{cases}=\begin{cases}2F_X(x) & x<0.5\\ 1 & x\ge 0.5\end{cases}$$

$$\Rightarrow f_X\left(x\mid X\le 0.5\right)=\frac{d}{dx}F_X\left(x\mid X\le 0.5\right)=\begin{cases}2f_X(x) & x<0.5\\ 0 & x\ge 0.5\end{cases}$$

$$f_X\left(x\mid X\le 0.5\right)$$