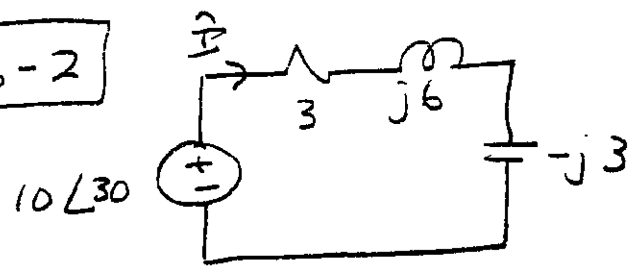
$\boxed{6.6\text{-}1}$ (a) $P_{AV} = \frac{1}{2} \times 10 \times 2 \cos(30+60) = \boxed{0\,W}$

(b) $P_{AV} = \frac{1}{2} \times 20 \times 5 \cos(30-45) = \boxed{48.3\,W}$

(c) $P_{AV} = \frac{1}{2} \times 8 \times 5 \cos(-35+80) = \boxed{14.14\,W}$

(d) $P_{AV} = \frac{1}{2} \times 25 \times 10 \cos(45-60) = \boxed{120.74\,W}$

$\boxed{6.6\text{-}2}$

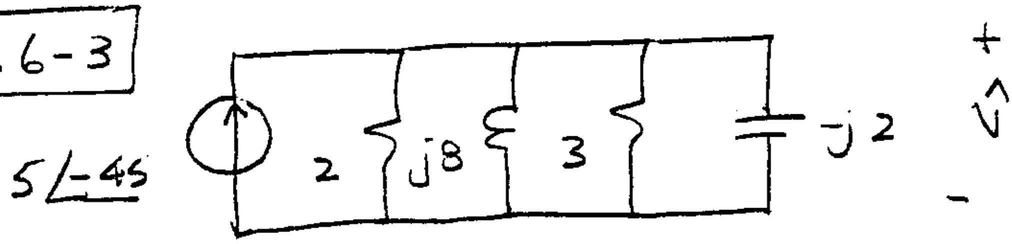


$$\hat{I} = \frac{10\angle 30}{3+j6-j3} = \frac{10}{3\sqrt{2}} \angle -15^\circ$$

$$P_{AV} = \frac{1}{2} \times 10 \times \frac{10}{3\sqrt{2}} \cos(30+15) = \boxed{8.33\,W}$$

$$P_{AV} = \frac{1}{2} |\hat{I}|^2 \, 3 = 8.33\,W$$ ✓

$\boxed{6.6\text{-}3}$



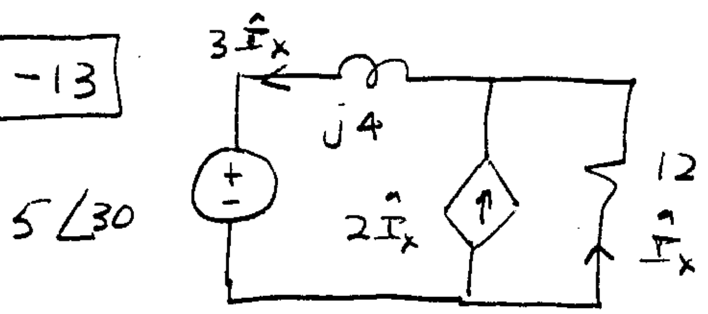
$$\hat{V} = \frac{5\angle -45}{\frac{1}{2} + \frac{1}{j8} + \frac{1}{3} + \frac{1}{-j2}} = \frac{5\angle -45}{\frac{5}{6} + j\frac{3}{8}} = 5.47\angle -69.23$$

$$P_{AV} = \frac{1}{2} \times 5 \times 5.47 \cos(-45+69.23) = \boxed{12.47\,W}$$

$$P_{AV_{2\Omega}} = \frac{1}{2} \frac{|\hat{V}|^2}{2} = 7.48\,W \qquad P_{AV_{3\Omega}} = \frac{1}{2} \frac{|\hat{V}|^2}{3} = 4.99$$

$$P_{AV} = P_{AV_{2\Omega}} + P_{AV_{3\Omega}}$$ ✓

$\boxed{6.6-13}$



$-$

$\hat{V} = 12\hat{I}_x$

$+$

$$5\angle 30 = -j4(3\hat{I}_x) - 12\hat{I}_x \quad \therefore \hat{I}_x = -\frac{5\angle 30}{12+j12}$$

$$= -0.29\angle -15$$

$$P_{AV} = -\frac{1}{2} \times 5 \times 3(-0.29)\cos(30+15)$$

$$= \boxed{1.56\,W} = P_{AV_{12\Omega}} = \frac{1}{2}|\hat{I}_x|^2 12 = 0.5\,W$$

$$+ P_{AV_{\text{cont source}}} = \frac{1}{2} \times |2\hat{I}_x||\hat{V}|\cos(-15+15) = 1.01\,W \quad \checkmark$$

$\boxed{6.6-14}$

$$V_{rms} = \sqrt{\frac{1}{2}\left[\int_0^1 (5t)^2 dt + \int_1^2 (5)^2 dt\right]}$$

$$= \sqrt{\frac{1}{2}\left[\frac{25}{3} + 25\right]} = \boxed{4.08\,V}$$

$$P_{AV} = \frac{V_{rms}^2}{2} = \boxed{8.33\,W}$$

$\boxed{6.6-15}$

$$V_{rms} = \sqrt{\frac{1}{4}\left[\int_0^2 (2t)^2 dt + \int_2^4 (-2t+8)^2 dt\right]}$$

$$= \sqrt{\frac{1}{4}\left[\frac{4t^3}{3}\Big|_0^2 + \left(\frac{4t^3}{3} - \frac{32t^2}{2} + 64t\right)_2^4\right]}$$

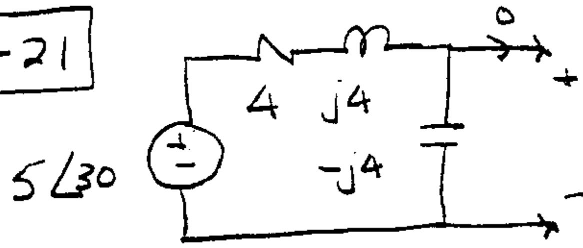
$$P_{AV} = \frac{V_{rms}^2}{2}$$

$$= \boxed{2.67\,W}$$

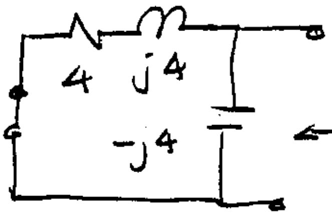
$$= \sqrt{\frac{1}{4}\left[\frac{32}{3} + (85.33 - 74.67)\right]}$$

$$= \boxed{2.31\,V}$$

$\boxed{6.6\text{-}21}$

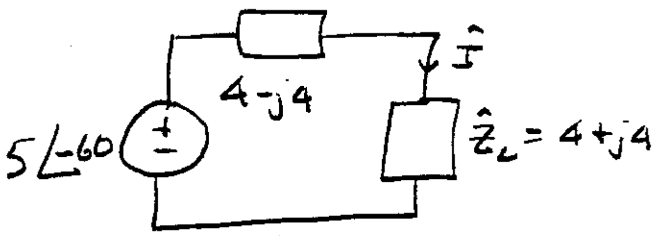


$$\hat{V}_{oc} = \frac{-j4}{4+j4-j4}\, 5\angle 30 = 5\angle -60^\circ$$



$$\hat{Z}_{TH} = \frac{(4+j4)(-j4)}{4+j4-j4} = 4-j4$$

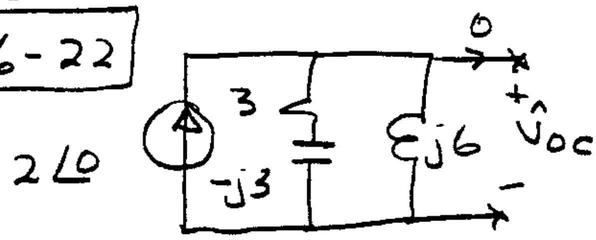
$$\therefore\ \hat{Z}_L = \hat{Z}_{TH}^{*} = \boxed{4+j4\ \Omega}$$



$$\hat{I} = \frac{5\angle -60}{8}$$

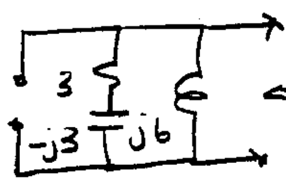
$$P_{AV} = \frac{1}{2}|\hat{I}|^2\, 4 = \boxed{0.78\ \text{W}}$$

$\boxed{6.6\text{-}22}$

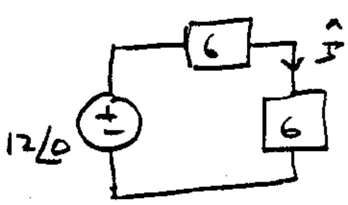


$$\hat{V}_{oc} = 2\angle 0 \times 6\angle 0 = 12\angle 0$$

$$\frac{(3-j3)(j6)}{3-j3+j6} = \frac{18+j18}{3+j3} = 6\angle 0$$



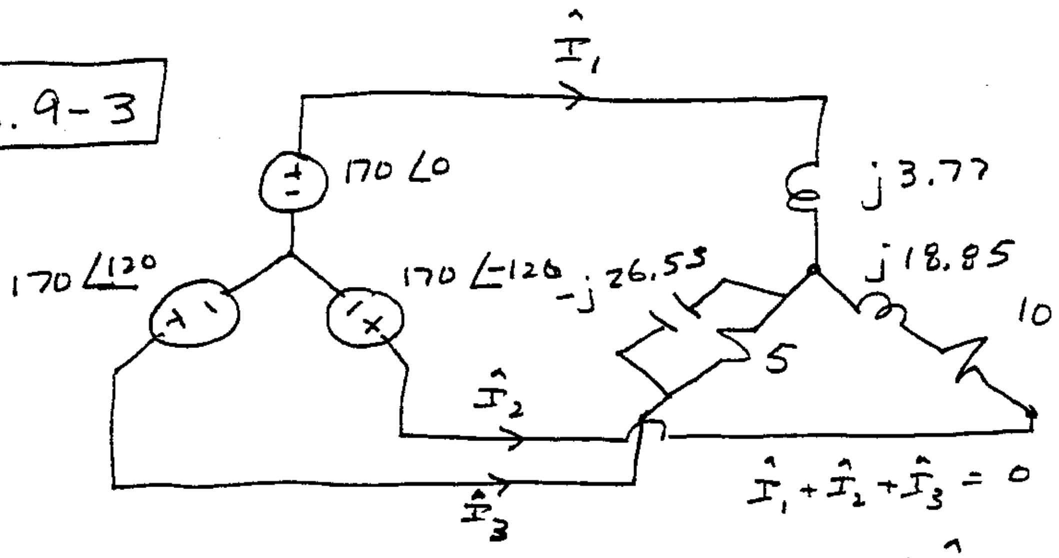
$$\hat{Z}_{TH} = 6\angle 0 \qquad \therefore\ \hat{Z}_L = \hat{Z}_{TH}^{*} = \boxed{6\angle 0}$$



$$\hat{I} = \frac{12}{12} = 1$$

$$P_{AV} = \frac{1}{2}|\hat{I}|^2\, 6 = \boxed{3\ \text{W}}$$

$\boxed{6.9-3}$



$\hat{I}_1 + \hat{I}_2 + \hat{I}_3 = 0$

$170\angle 0 - 170\angle 120 = j3.77\,\hat{I}_1 - (10+j18.85)\,\hat{I}_2$

$170\angle 0 - 170\angle 120 = j3.77\,\hat{I}_1 - \underbrace{5 \| (-j26.53)}_{4.91\angle -10.67}\,\cancel{\hat{I}_3}\ \ -\hat{I}_1 - \hat{I}_2$

$= 4.83 - j0.91$

$\therefore\ j3.77\,\hat{I}_1 - 21.34\angle 62.05\,\hat{I}_2 = 294.45\angle 30$

$(4.83 + j2.86)\hat{I}_1 + (4.83 - j0.91)\hat{I}_2 = 294.45\angle -30$

$$\hat{I}_1 = \frac{\begin{vmatrix} 294.45\angle 30 & -21.34\angle 62.05 \\ 294.45\angle -30 & (4.83 - j0.91) \end{vmatrix}}{\begin{vmatrix} j3.77 & -21.34\angle 62.05 \\ (4.83+j2.86) & (4.83-j0.91) \end{vmatrix}} = 26.89 - j48.96 = 55.86\angle -61.22^\circ$$

$= 3.94\angle 151.03$

$\hat{I}_2 = -3.45 + j1.91\,A$

$\hat{I}_3 = -\hat{I}_1 - \hat{I}_2 = -23.44 + j47.05 = A$ $\quad 52.57\angle 116.48$

$$\boxed{\begin{aligned} i_1(t) &= 55.86\sin(120\pi t - 61.22^\circ)\ A \\ i_2(t) &= 3.94\sin(120\pi t + 151.03^\circ)\ A \\ i_3(t) &= 52.57\sin(120\pi t + 116.48)\ A \end{aligned}}$$

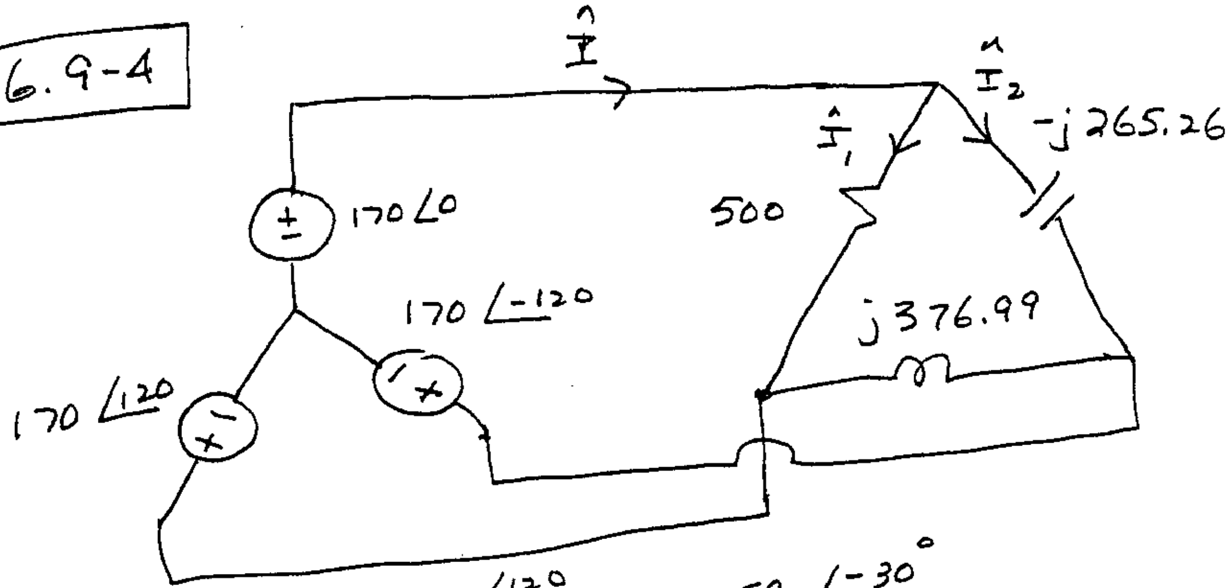
$P_{AV_1} = \frac{1}{2}\,170 \times 55.86\cos(0 + 61.22)$
$= 2285.96\,W$

$P_{AV_2} = \frac{1}{2}\,170 \times 3.94\cos(-120 - 151.03)$
$= 6.02\,W$

$P_{AV_3} = \frac{1}{2}\,170 \times 52.57\cos(120 - 116.48) = 4460.02\,W$

$P_{AV} = \boxed{6752 \quad W}$

$\boxed{6.9\text{-}4}$



$$\hat{I}_1 = \frac{170\angle 0 - 170\angle 120^\circ}{500} = 0.59\angle -30^\circ$$

$$\hat{I}_2 = \frac{170\angle 0 - 170\angle -120^\circ}{-j265.26} = \frac{294.45\angle 30}{265.26\angle -90}$$

$$= 1.11\angle 120^\circ$$

$$\hat{I} = \hat{I}_1 + \hat{I}_2$$

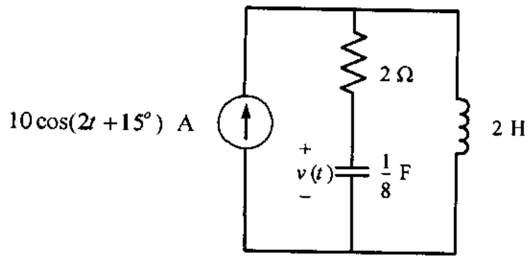
$$= 0.67\angle 93.78^\circ$$

$$\therefore \boxed{i(t) = 0.67\cos(120\pi t + 93.78^\circ)\ \text{A}}$$

$$P_{AV} = \frac{1}{2}|\hat{I}_1|^2 500$$

$$= \boxed{87.03\ \text{W}}$$

Extra Problem



Using the complex power concept find the followings:

(a) The complex power absorb by the load ?

(b) The Average power absorb by the load ?

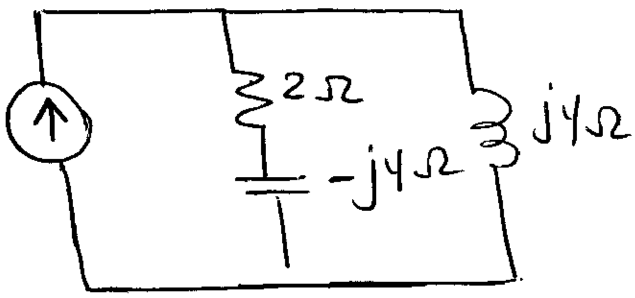
(c) Power factor of the load ?

(d) verify $\theta_{\text{Complex Power}} = \theta_{\text{power factor}} = \theta_{\text{Impedence}}$

<u>solution</u>

(a)

$$Z_L = (j4) \| (2-j4)$$



$$= \frac{j4(2-j4)}{j4+2-j4}$$

$$= 8 + j4 \ \Omega$$

$$S_{Load} = \frac{1}{2} Z_{Load} \left| 10\angle 15^\circ \right|^2 = \frac{1}{2}(8+j4)(10)^2$$

$$= 400 + j200 \quad VA$$

(b) $P_{Load} = \mathrm{Re}[S_{Load}] = 400 \text{ W}$

(c) $\theta_{S_{Load}} = \tan^{-1}\left(\frac{200}{400}\right) = 26.56^\circ$

$pf = \cos(26.56) = 0.89$ lagging

(d) $\theta_Z = \tan^{-1}\left(\frac{4}{8}\right) = 26.56^\circ = \theta_{S_{Load}} = \theta_{pf}$