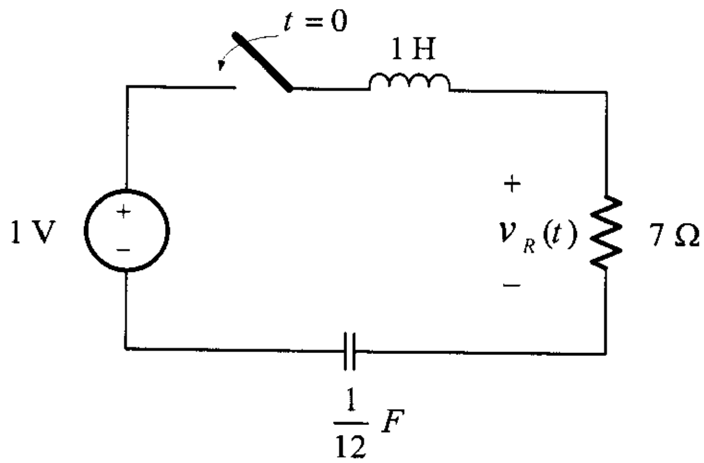
EE 207-01 – Winter 2010
Quiz 6

SER	ID	NAME **KEY**

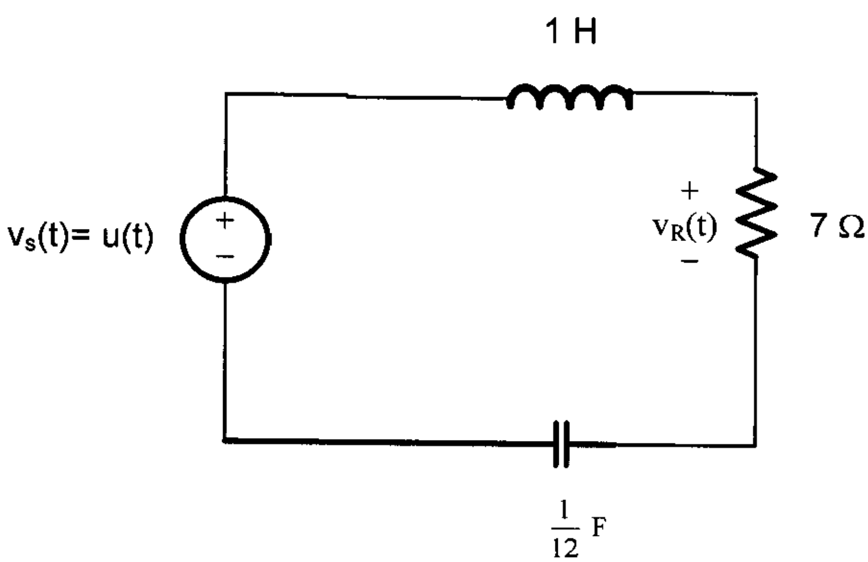
Let the switch on the circuit shown below close at t = 0



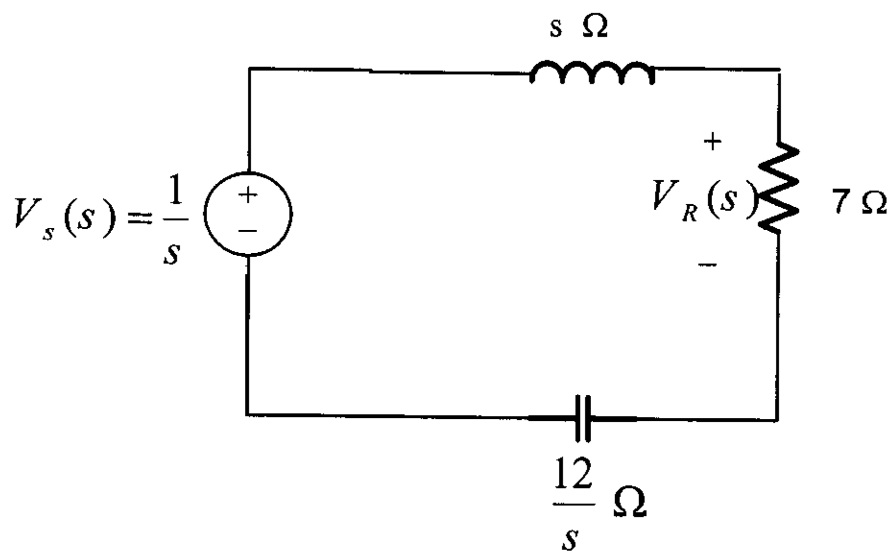
Assuming that the inductor and the capacitor has zeros initial conditions. Using the Laplace transform method , find the voltage across the resistant $V_R(t)$

Solution

We can model the switch as a step function u(t) as follows:



We now apply the Lapalce transform to the circuit to obtain



Using voltage division we have

$$V_R(s) = \frac{7}{s + 7 + \frac{12}{s}} V_s(s) = \frac{7}{s + 7 + \frac{12}{s}}\left(\frac{1}{s}\right)$$

$$= \frac{7}{s^2 + 7s + 12} = \frac{7}{(s+3)(s+4)}$$

using partial fraction expansion we have

$$V_R(s) = \frac{7}{(s+3)(s+4)} = \frac{A_1}{(s+3)} + \frac{A_2}{(s+4)}$$

$$= \frac{7}{(s+3)} - \frac{7}{(s+4)}$$

Therefore

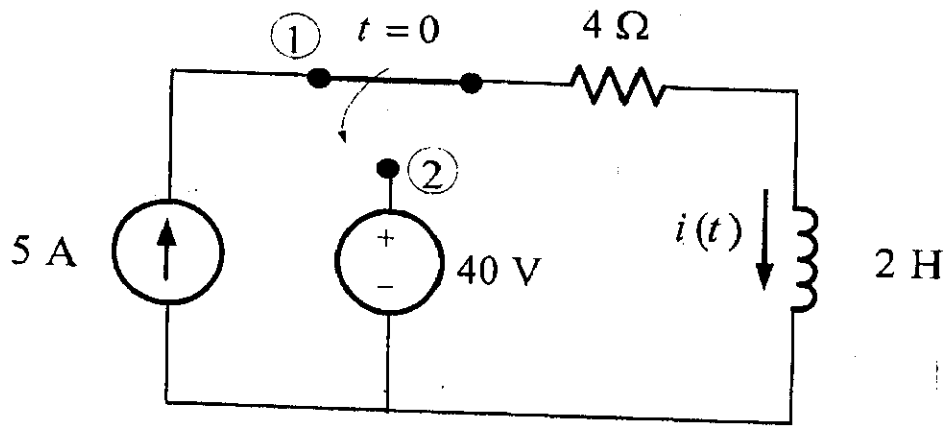
$$v_R(t) = 7e^{-3t}u(t) - 7e^{-4t}u(t)$$

$$= 7(e^{-3t} - e^{-4t})\mathrm{u(t)}$$

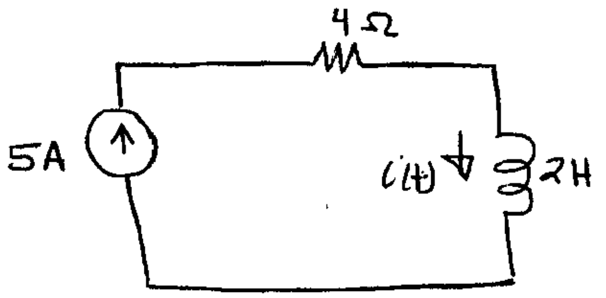
EE 207-02 – Winter 2010
Quiz 6

SER	ID	NAME KEY

For the circuit shown below, the switch was in position 1 for a long time then at $t = 0$, the switch was moved to position 2, using **Laplace Transform Method** find $i(t)$ for all t?



For $t<0$

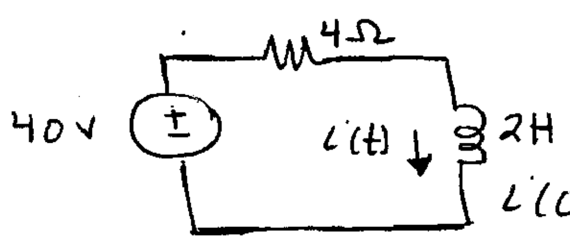


(short for DC)

$i(t) = 5A$

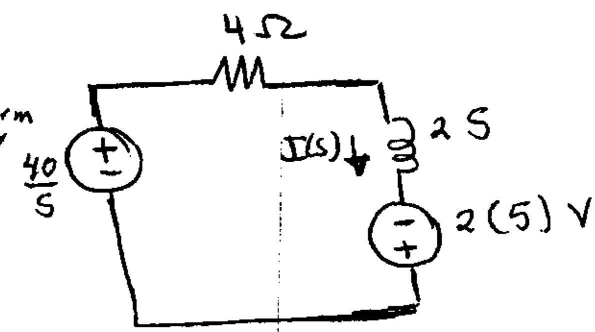
$\Rightarrow i(0^-) = 5A$

For $t<0$



$i(0^-) = 5A$

Laplace Transform $\Longrightarrow$



KVL $\quad -\frac{40}{S} + 4I(s) + 2SI(s) - 10 = 0$

$$\rightarrow I(s) = \frac{40/s}{(2s+4)} + \frac{10}{2s+4} = \frac{20}{s(s+2)} + \frac{5}{(s+2)}$$

$$= \frac{20+5s}{s(s+2)}$$

(continue) $\rightarrow$

$$I(s) = \frac{20+5s}{s(s+2)} = \frac{A_1}{s} + \frac{A_2}{(s+2)}$$

$$A_1 = \left.\frac{20+5s}{(s+2)}\right|_{s=0} = \frac{20}{2} = 10$$

$$A_2 = \left.\frac{20+5s}{s}\right|_{s=-2} = \frac{20-10}{-2} = -5$$

$$I(s) = \frac{10}{s} - \frac{5}{(s+2)}$$

$$\Rightarrow \quad i(t) = 10\,u(t) - 5e^{-2t}u(t)$$