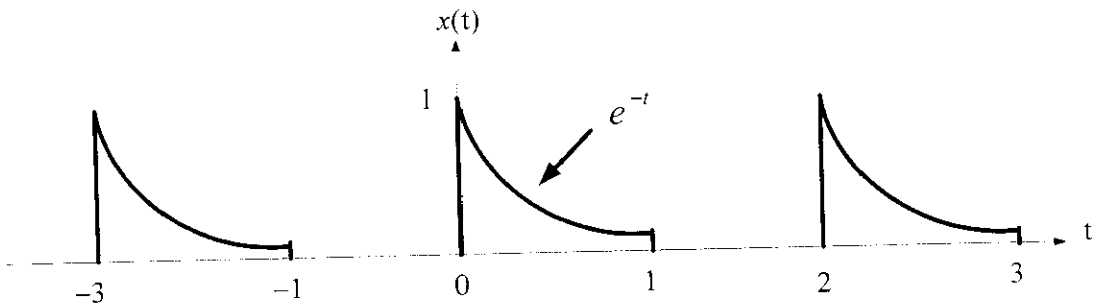
EE 207-01 – Winter 2010
Quiz 3

SER	ID	NAME KEY



$T_0 = 2$ s

$f_0 = \frac{1}{2}$ Hz

For the periodical signal x(t) shown above :

Find the complex Fourier Coefficients X(n) ?

$$X_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt = \frac{1}{2}\int_0^1 e^{-t}\,dt = \frac{1}{2}(1-e^{-1}) = 0.3161$$

$$X_n = \frac{1}{T_0}\int_{T_0} x(t)\,e^{-j2\pi n f_0 t}\,dt = \frac{1}{2}\int_0^1 e^{-t}\,e^{-j2\pi n \frac{1}{2}t}\,dt$$

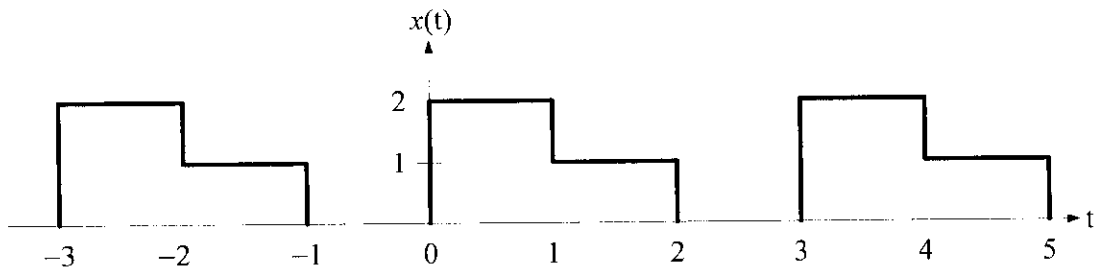
$$= \frac{1}{2}\int_0^1 e^{-(1+j\pi n)t}\,dt = -\frac{1}{2}\frac{1}{(1+j\pi n)}\,e^{-(1+j\pi n)t}\Big|_0^1$$

$$= -\frac{1}{2}\frac{1}{(1+j\pi n)}e^{-(1+j\pi n)} + \frac{1}{2}\left(\frac{1}{1+j\pi n}\right)$$

$$= \frac{1}{2}\frac{1}{(1+j\pi n)}\left[1 - e^{-(1+j\pi n)}\right]$$

EE 207-02 – Winter 2010
Quiz 3

SER	ID	NAME



$T_0 = 3$ s

$f_0 = \frac{1}{3}$ Hz

For the periodical signal x(t) shown above :

Find the complex Fourier Coefficients X(n) ?

$$X_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt = \frac{1}{3}\left((1)(2)+(1)(1)\right) = 1$$

$$X_n = \frac{1}{T_0}\int_{T_0} x(t)\,e^{-j\frac{2\pi nt}{T_0}}\,dt =$$

$$= \frac{1}{3}\left[\int_0^1 2\,e^{-j\frac{2\pi nt}{3}}\,dt + \int_1^2 e^{-j\frac{2\pi nt}{3}}\,dt\right]$$

$$= \frac{1}{3}\left[-\frac{2}{j\frac{2\pi n}{3}}\,e^{-j\frac{2\pi nt}{3}}\Big|_0^1 - \frac{1}{j\frac{2\pi n}{3}}\,e^{-j\frac{2\pi nt}{3}}\Big|_1^2\right]$$

$$= -\frac{1}{j\pi n}\left[e^{-j\frac{2\pi n}{3}} - 1\right] - \frac{3}{j2\pi n}\left[e^{-j\frac{4\pi n}{3}} - e^{-j\frac{2\pi n}{3}}\right]$$

continue ⟶

$$X_n = -\frac{1}{j\pi n}\left[e^{-j\frac{\pi n}{3}} - e^{j\frac{\pi n}{3}}\right]e^{-j\frac{\pi n}{3}}$$

$$- \frac{1}{j2\pi n}\left[e^{-j\frac{4\pi n}{3}} - e^{-j\frac{2\pi n}{3}}\right]$$

$$= \frac{2}{\pi n}\sin\frac{\pi n}{3}\, e^{-j\frac{\pi n}{3}} - \frac{1}{j2\pi n}\left[e^{-j\frac{4\pi n}{3}} - e^{-j\frac{2\pi n}{3}}\right.$$