KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

SEMESTER 092

EE 207 MAJOR EXAM I

DATE: WENSDAY 31-3-2010

TIME: 4:30-6:30 PM

SER	ID	NAME KEY	SECTION

	Maximum Score	Score
Problem 1	10	
Problem 2	15	
Problem 3	15	
Problem 4	20	
TOTAL	60	

Problem 1 (10)

(a) For the input x(t) and output y(t) given as **(3)**

y(t) = x(t) + 5

Is the system is Linear or Non linear , *Explain* ?

(b) For the input x(t) and output y(t) given as **(3)**

$$y(t) = x(t^2)$$

Is the system is time invariant or time variant , *Explain* ?

(c) Evaluate $\delta(t-4)e^{-2(t-3)}$? **(2)**

(d) Evaluate $\delta(t-4) * e^{-2(t-3)}$ were * is the convolution operator ? **(2)**

(a) For input $x_1(t) \Rightarrow y_1(t) = \mathcal{H}[x_1(t)] = x_1(t) + 5$

For input $x_2(t) \Rightarrow y_2(t) = \mathcal{H}[x_2(t)] = x_2(t) + 5$

For input $\alpha_1 x_1(t) + \alpha_2 x_2(t) \Rightarrow \mathcal{H}[\alpha_1 x_1(t) + \alpha_2 x_2(t)]$

$= \alpha_1 x_1(t) + \alpha_2 x_2(t) + 5$

However $\alpha_1 y_1(t) + \alpha_2 y_2(t) = \alpha_1 x_1 + \alpha_2 x_2 + 5(\alpha_1 + \alpha_2)$

$\neq \mathcal{H}[\alpha_1 x_1 + \alpha_2 x_2]$

$\Rightarrow$ system is not linear

(b) $y(t) = \mathcal{H}[x(t)] = x(t^2)$

Let $x_1(t) = x(t-\alpha)$; $\Rightarrow \mathcal{H}[x_1(t)] = x_1(t^2)$

$\Rightarrow \mathcal{H}[x(t-\alpha)] = x(t^2-\alpha)$

However $y(t-\alpha) = x(t-\alpha)^2$

$\Rightarrow$ $\mathcal{H}[x(t-\alpha)] \neq y(t-\alpha)$

$\Rightarrow$ system is time variant

(c) $\delta(t-4)\, e^{-2(t-3)} = e^{-2(4-3)}\, \delta(t-4)$

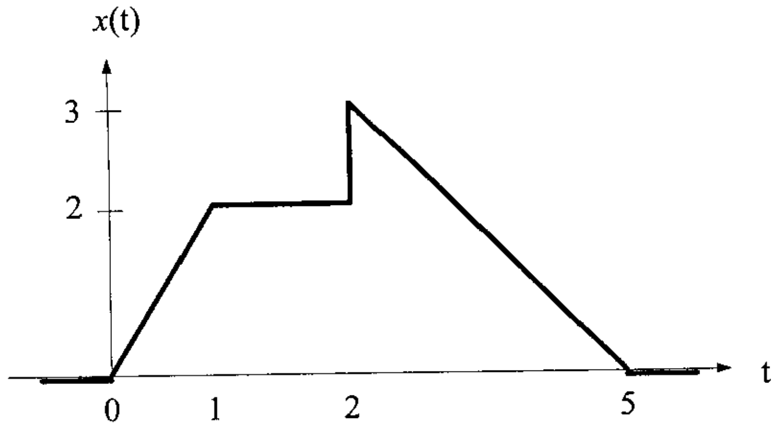
$= e^{-2}\, \delta(t-4)$

(d) $\delta(t-4) * e^{-2(t-3)} = e^{-2(t-3-4}$

$= e^{-2(t-7)}$

Problem 2 (15)

Let x(t) be a signal as shown below



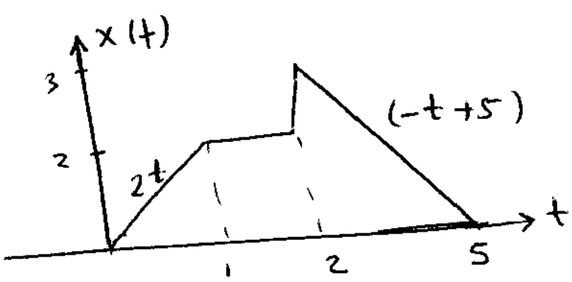
(a) Expand x(t) in terms of the singularities functions ? **(7)**

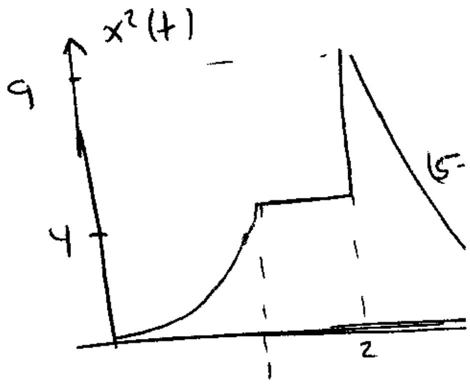
(b) Calculate the energy and average power of x(t) ? **(6)**

(c) Is x(t) an energy or a power signal ? *Explain* ? **(2)**

(a) $x(t) = 2r(t) - 2r(t-1) + u(t-2) - r(t-2) + r(t-5)$

(b)





$$E = \int_0^1 4t^2\,dt + \int_1^2 2^2\,dt + \int_2^5 (5-t)^2\,dt$$

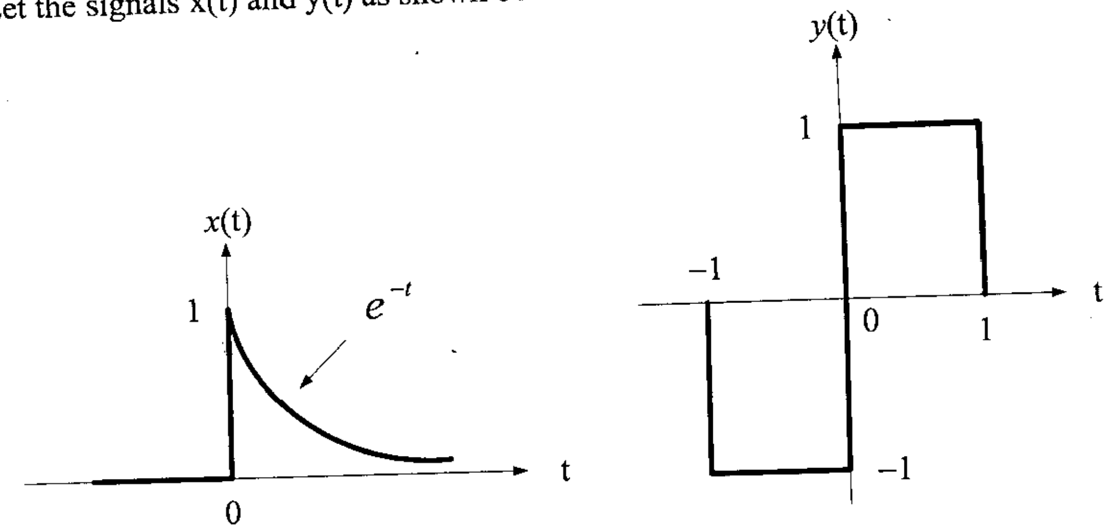
$$= \frac{4}{3} + 4 + 9 = \frac{43}{3} = 14\tfrac{1}{3} \quad J$$

$P = 0$

(c) Since $E < \infty \Rightarrow$ x(t) is an energy si

Problem 3 (15)

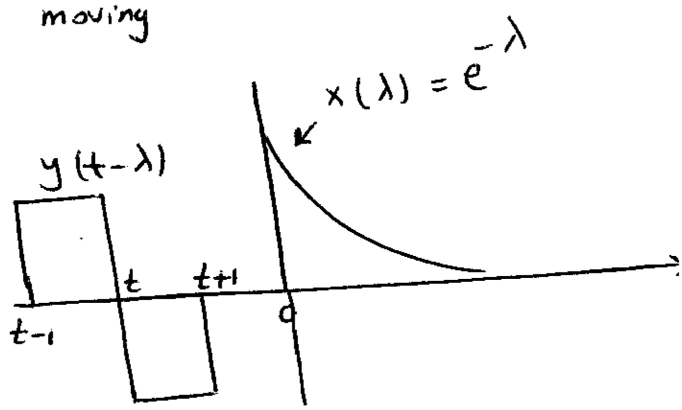
Let the signals x(t) and y(t) as shown below



Evaluate the convolution integral $x(t)*y(t)$?

Solution 1

$$x(t) * y(t) = \int_{-\infty}^{\infty} \underbrace{x(\lambda)}_{\text{Fix}} \, \underbrace{y(t-\lambda)}_{\text{moving}} \, d\lambda$$



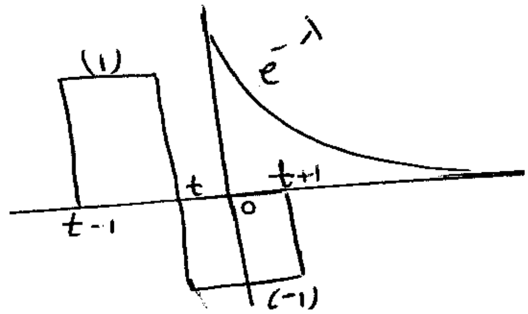
For $t+1<0 \Rightarrow t<-1$

$x(t) * y(t) = 0$ (No overlapping)

For $0<t+1<1 \Rightarrow -1<t<0$

$$x(t)*y(t) = \int_0^{t+1} (-1)\, e^{-\lambda} \, d\lambda$$

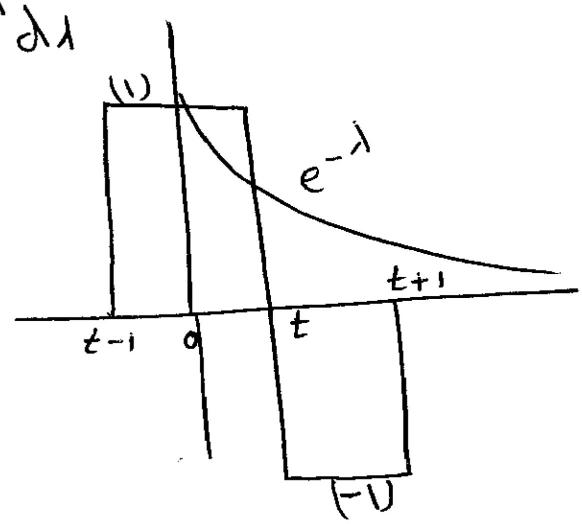
$$= e^{-(t+1)} - 1$$



For $1 < t+1 < 2 \Rightarrow 0 < t < 1$

$$x(t) * y(t) = \int_0^t (1)e^{-\lambda} + \int_t^{t+1} (-1)e^{-\lambda}\, d\lambda$$

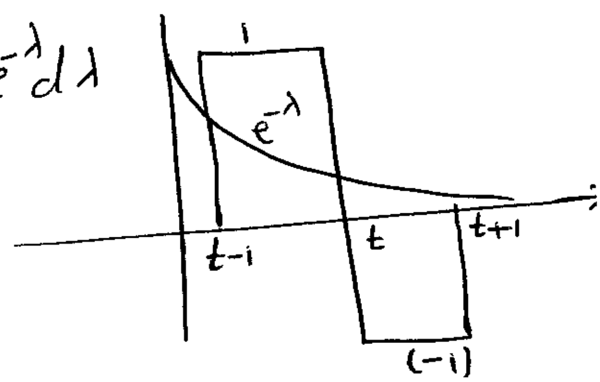
$$= 1 - 2e^{-t} + e^{-(t+1)}$$



For $t+1 > 2 \Rightarrow t > 1$

$$x(t) * y(t) = \int_{t-1}^t (1)e^{-\lambda}\, d\lambda + \int_t^{t+1} (-1)e^{-\lambda}\, d\lambda$$

$$= e^{(1-t)} - 2e^{-t} + e^{-(1+t)}$$



~~another solution~~ $\Rightarrow$

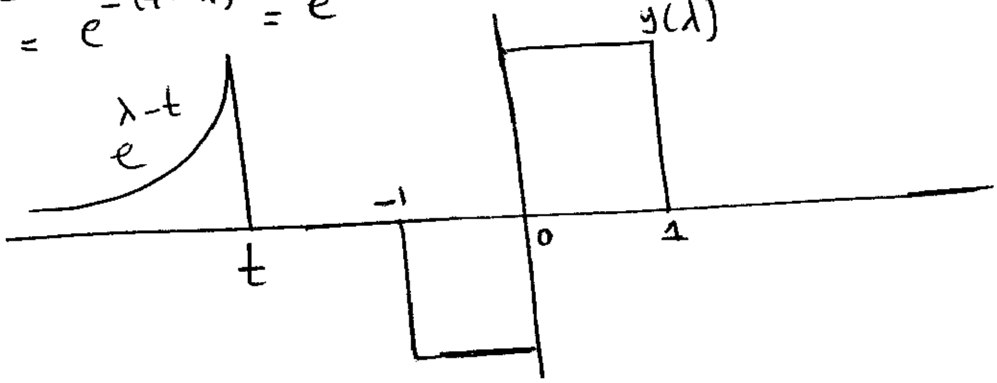
$$x(t) * y(t) = \begin{cases} 0 & t < - \\ e^{-(t+1)} - 1 & -1 < t < \\ 1 - 2e^{-t} + e^{-(t+1)} & 0 < t < \\ e^{(1-t)} - 2e^{-t} + e^{-(1+t)} & t > 1 \end{cases}$$

solution 2

$$y(t) * x(t) = \int_{-\infty}^{\infty} \underset{\substack{\downarrow \\ \text{Fix}}}{y(\lambda)}\; \underset{\substack{\downarrow \\ \text{moving}}}{x(t-\lambda)}\, d\lambda$$

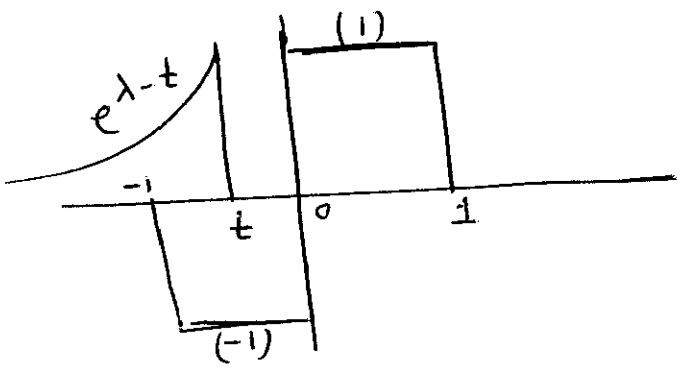
$$x(\lambda) = e^{-\lambda}$$

$$x(t-\lambda) = e^{-(t-\lambda)} = e^{\lambda-t}$$



For $t < -1$ $\quad y(t) * x(t) = 0$ (No overlapp.

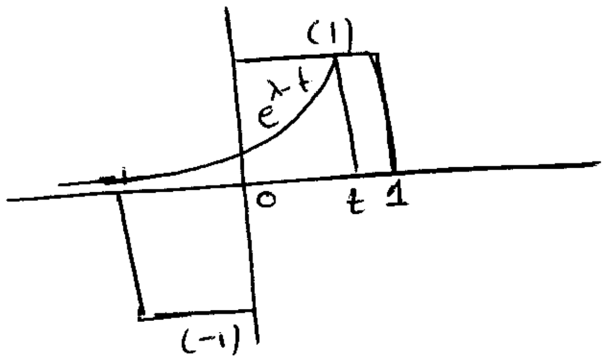
For $-1 < t < 0$



$$y(t) * x(t) = \int_{-1}^{t} (-1)\, e^{\lambda-t}\, d\lambda$$

$$= e^{-(t+1)} - 1$$

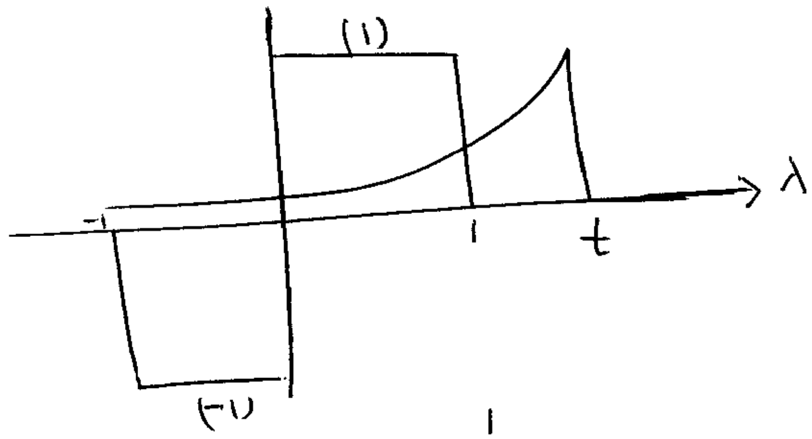
For $0 < t < 1$



$$y(t) * x(t) = \int_{-1}^{0} (-1)\, e^{\lambda-t}\, d\lambda + \int_{0}^{t} (1)\, e^{\lambda-t}\, d\lambda$$

$$= 1 - 2e^{-t} + e^{-(t+1)}$$

For $t > 1$



$$y(t) * x(t) = \int_{-1}^{0} (-1)\, e^{\lambda - t}\, d\lambda + \int_{0}^{1} (1)\, e^{\lambda - t}\, d\lambda$$

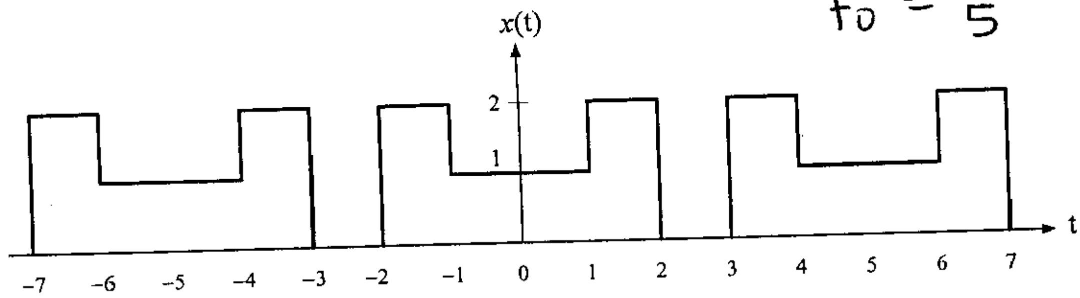
$$= e^{(1-t)} - 2e^{-t} + e^{-(t+1)}$$

Problem 4 (20)

Let x(t) be a periodical function shown below

$T_0 = 5$

$f_0 = \frac{1}{5}$



(a) Find the trigonometric Fourier series coefficient a_n and b_n for all n ? **(16)**

(b) Evaluate the coefficient a_n and b_n associated with the harmonic of frequency 1 Hz ? **(4)**

(a) x(t) is an even function $\Rightarrow$ $b_n = 0$

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt = \frac{1}{5}\left[(1)(2) + (2)(1) + (1)(2)\right]$$

$$= \frac{6}{5}$$

$$a_n = \frac{2}{T_0}\int_{T_0} x(t)\ \cos 2\pi n f_0 t$$

$$= \frac{2}{5}\left[\int_{-2}^{-1}(2)\cos\frac{2\pi n}{5}t\,dt + \int_{-1}^{1}(1)\cos\frac{2\pi n t}{5}dt + \int_{1}^{2}(2)\cos\frac{2\pi n t}{5}dt\right]$$

$$= \frac{2}{5}\left[2\frac{5}{2\pi n}\sin\frac{2\pi n t}{5}\Big|_{-2}^{-1} + \frac{5}{2\pi n}\sin\frac{2\pi n t}{5}\Big|_{-1}^{1} + 2\frac{5}{2\pi n}\sin\frac{2\pi n t}{5}\Big|_{1}^{2}\right]$$

Continue $\Rightarrow$

$$a_n = \frac{1}{\pi n}\left[4 \sin \frac{4\pi n}{5} - 2 \sin \frac{2\pi n}{5}\right]$$

(b) Since $T_0 = 5 \Rightarrow f_0 = \frac{1}{5}$ Hz

harmonic of 1 Hz $\Rightarrow n f_0 = 1$

$\Rightarrow \frac{n}{5} = 1 \Rightarrow n = 5$

$$a_5 = \frac{1}{\pi(5)}\left[4 \sin \frac{4\pi(5)}{5} - 2 \sin \frac{2\pi(5)}{5}\right]$$

$a_5 = 0$

$b_5 = 0$ ($b_n = 0$ for all n).