KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

SEMESTER 092

EE 207 MAJOR EXAM II

DATE: Saturday 15-5-2010

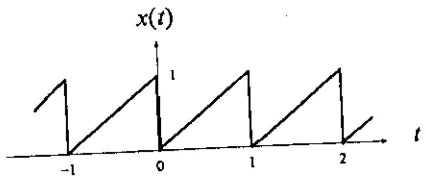
TIME: 7:00-8:30 PM

SER	ID	NAME KEY	SECTION

	Maximum Score	Score
Problem 1	**20**	
Problem 2	**25**	
Problem 3	**15**	
TOTAL	**60**	

Problem 1 (20)

Let $x(t)$ be a periodical signal as shown below



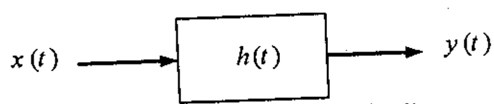
$T_0 = 1 \Rightarrow f_0 = 1$ Hz

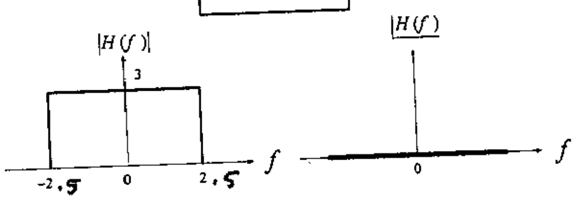
If the complex Fourier coefficients of $x(t)$ is given as $X_n = j\dfrac{1}{2\pi n}$ $\quad n \neq 0$

(a) Find X_0 ? (5)

(b) Compute $\sum_{n=-\infty}^{\infty} |X_n|^2$ (5)

If $x(t)$ above is passed through a low pass filter with transfer function $H(f)$ as shown below





(c) what is y(t) ? (write a mathematical expression , **do not use any convolution expression**) (5)

(b) Find the power of the output signal y(t) (from part c) ? (5)

(a) $X_0 = \dfrac{1}{T_0}\displaystyle\int_{T_0} x(t)\,dt = \dfrac{1}{1}\int_0^1 t\,dt = (\tfrac{1}{2})(1) = \dfrac{1}{2}$

(b) $\displaystyle\sum_{n=-\infty}^{\infty} |X_n|^2 = \frac{1}{T_0}\int_{T_0} x^2(t)\,dt$ (Parseval's Thm)

$\Rightarrow \displaystyle\sum_{n=-\infty}^{\infty} \left|j\frac{1}{2\pi n}\right|^2 = \sum_{n=-\infty}^{\infty} \frac{1}{(2\pi n)^2} = \frac{1}{1}\int_0^1 t^2\,dt = \frac{1}{3}$

(c) Sinc the low pass filter pass only harmonics with frequencies ≤ 2.5 HZ

Since $f_0 = 1$ HZ

$\Rightarrow$ only frequencies 0 (DC), 1, 2 are passed

$\Rightarrow n = 0, 1, 2$

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{j2\pi n f_0 t}$$

$$y(t) = \sum_{n=-\infty}^{\infty} \underbrace{X_n H(nf_0)}_{Y_n} e^{j2\pi n f_0 t}$$

$Y_n = X_n H(nf_0) \qquad n = 0, \pm1, \pm2$

$Y_n = 0 \qquad |n| > 2$

$Y_0 = X_0 H(0) = \frac{1}{3}(3) = 1$

$Y_1 = X_1 H(f_0) = X_1 H(1) = 3X_1 = 3\left(j\frac{1}{2\pi}\right) = \frac{3}{2\pi}\angle 90^\circ$

$Y_{-1} = Y_1^* = \frac{3}{2\pi}\angle -90^\circ$

$Y_2 = X_2 H(2f_0) = X_2 H(2) = 3X_2 = 3\left(j\frac{1}{4\pi}\right) = \frac{3}{4\pi}\angle 90^\circ$

$Y_{-2} = Y_2^* = \frac{3}{4\pi}\angle -90^\circ$

$$\Rightarrow y(t) = \sum_{n=-2}^{2} Y_n e^{j2\pi n(1)t}$$

$$= Y_{-2} e^{-j4\pi t} + Y_{-1} e^{-j2\pi t} + Y_0 + Y_1 e^{j2\pi t} + Y_2 e^{j4\pi t}$$

<u>Since</u> $Y_{-n} = Y_n^*$

$$\Rightarrow Y_{-n} = |Y_n| \angle -\theta_n$$

$$\Rightarrow Y_{-n} e^{-j2\pi n f_0 t} + Y_n e^{j2\pi n f_0 t}$$

$$= |Y_n| e^{-j\theta_n} e^{-j2\pi n f_0 t} + |Y_n| e^{j\theta_n} e^{j2\pi n f_0 t}$$

$$= |Y_n| e^{-j(2\pi n f_0 t + \theta_n)} + |Y_n| e^{j(2\pi n f_0 t + \theta_n)}$$

$$= 2|Y_n| \cos(2\pi n f_0 t + \theta_n)$$

$$\Rightarrow y(t) = 2|Y_2| \cos(2\pi 2t + \theta_2) + Y_0$$

$$+ 2|Y_1| \cos(2\pi t + \theta_1)$$

$$= 2\left(\frac{3}{4\pi}\right) \cos(4\pi t + 90^\circ) + 1 + 2\left(\frac{3}{2\pi}\right) \cos(2\pi t + 90^\circ)$$

$$= 1 - \frac{3}{2\pi} \sin(4\pi t) - \frac{3}{\pi} \sin(2\pi t)$$

(c)

$$P_y = \sum_{n=-\infty}^{\infty} |Y_n|^2$$

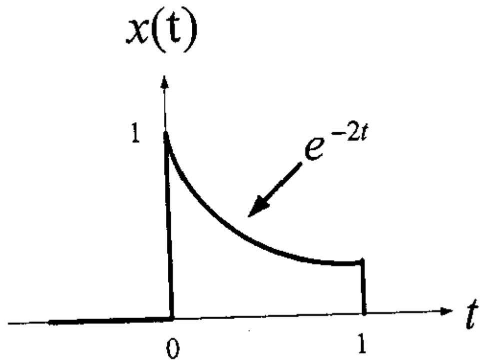
$$= Y_0^2 + 2\sum_{n=1}^{\infty} |Y_n|^2$$

$$= (1)^2 + 2\left[\left(\frac{3}{2\sqrt{3}}\right)^2 + \left(\frac{3}{4\sqrt{3}}\right)^2\right]$$

$$= 1.57 \quad W$$

Problem 2 (25)

Let $x(t)$ be a signal as shown below



(a) Using the integration formula only ,find the Fourier Transform of $x(t)$? **(18)**

(b) If the signal $x(t)$ above , is passed through a system with differential equation given as

$$3\,\frac{d^2y(t)}{d^2t}+6\,\frac{dy(t)}{dt}+4\,y(t-1)=x(t)$$

Find the Fourier transform of y(t) , Y(s) ? (7)

(a)
$$X(f)=\int_{-\infty}^{\infty}x(t)\,e^{-j2\pi ft}$$
$$=\int_{0}^{1}e^{-2t}\,e^{-j2\pi ft}\,dt$$
$$=\int_{0}^{1}e^{-(2+j2\pi f)t}\,dt$$
$$=-\left.\frac{e^{-(2+j2\pi f)t}}{(2+j2\pi f)}\right|_{0}^{1}$$
$$=\frac{1-e^{-(2+j2\pi f)}}{(2+j2\pi f)}$$

(b) Taking Fourier Transform for both side

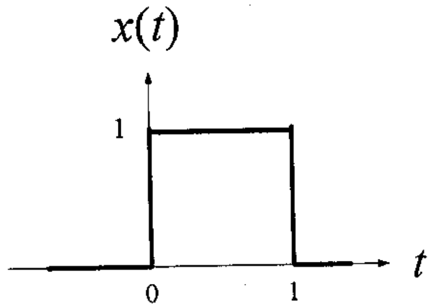
$$3(j2\pi f)^2 Y(f) + 6(j2\pi f)Y(f) + 4Y(f)e^{-j2\pi f} = X(f)$$

$$\Rightarrow Y(f) = \frac{X(f)}{3(j2\pi f)^2 + 6(j2\pi f) + 4e^{-j2\pi f}}$$

$$= \frac{1 - e^{-(2+j2\pi f)}}{\left[3(j2\pi f)^2 + 6(j2\pi f) + 4e^{-j2\pi f}\right]\left[2+j2\pi f\right]}$$

Problem 3 (15)

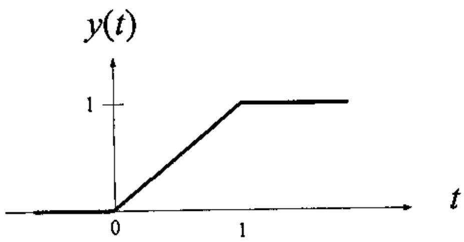
Let the signals $x(t)$ as shown below



(a) Show that Laplace Transform of $x(t)$ is given as (5)

$$X(s) = \frac{1 - e^{-s}}{s}$$

(b) Using the Laplace Transform Table and properties only (10)
find the Laplace Transform of $y(t)$, $Y(s)$?



(a) $$X(s) = \int_0^1 e^{-st}\, dt = -\frac{e^{-st}}{s}\Big|_0^1 = \frac{1 - e^{-s}}{s}$$

or $$x(t) = u(t) - u(t-1)$$

$$\Rightarrow X(s) = \frac{1}{s} - \frac{1}{s} e^{-s} = \frac{1 - e^{-s}}{s}$$

(b)

Since $$\underset{\text{Part (b)}}{y(t)} = \int_{-\infty}^{t} \underset{\text{Part (a)}}{x(t')}\, dt'$$

$$\Rightarrow Y(s) = \frac{X(s)}{s} + \frac{\cancelto{0}{\int_{-\infty}^{0} x(t')\,dt'}}{s}$$

$$= \frac{X(s)}{s} = \frac{1 - e^{-s}}{s^2}$$

<u>Solution (2)</u>

$$y(t) = r(t) - r(t-1)$$

$$r(t) = \int_{-\infty}^{t} u(t')\,dt' \xLeftrightarrow{\text{Laplace}} \frac{1/s}{s} + \frac{\cancelto{0}{\int_{-\infty}^{0} u(t')\,dt'}}{s}$$

$$= \frac{1}{s^2}$$

<u>or</u> from Table (5-3)

$$\frac{t^n e^{-\alpha t} u(t)}{n!} \Longleftrightarrow \frac{1}{(s+\alpha)^{n+1}}$$

$$\Rightarrow r(t) = t\,u(t) \Longleftrightarrow \frac{1}{s^2}$$

$$\Rightarrow Y(s) = \frac{1}{s^2} - \frac{1}{s^2}e^{-s}$$

$$= \frac{1 - e^{-s}}{s^2}$$

Using Direct Formula (For information only)

$$Y(s) = \int_0^\infty y(t)\, e^{-st}\, dt$$

$$= \int_0^1 t e^{-st} dt + \int_1^\infty e^{-st} dt$$

$$\underbrace{\int_0^1 t e^{-st} dt}_{\text{integrating by parts}} = -\frac{t e^{-st}}{s}\Big|_0^1 + \frac{1}{s}\int_0^1 e^{-st} dt$$

$$= -\frac{e^{-s}}{s} - \frac{e^{-s} - 1}{s^2}$$

$$\int_1^\infty e^{-st} dt = \int_0^\infty u(t-1)\, e^{-st} = \mathcal{L}[u(t-1)] = \frac{e^{-s}}{s}$$

$$\Rightarrow Y(s) = \frac{1 - e^{-s}}{s^2}$$