

EE 207 – Fall 2009  
Quiz 5

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Let  $y(t)$  be the out to Linear Time Invariant system, be described as the following differential equation

$$y''(t) + 4y'(t) + 3y(t) = e^{-5t}u(t)$$

$$y(0) = 1 \quad y'(0) = -1$$

Find the output  $y(t)$  ?

$$\frac{d^2y(t)}{dt^2} \leftrightarrow s^2 Y(s) - \underbrace{sy(0)}_1 - \underbrace{\frac{dy(t)}{dt}\bigg|_{t=0}}_{-1}$$

$$\frac{dy(t)}{dt} \leftrightarrow sY(s) - \underbrace{y(0)}_1$$

$$e^{-5t}u(t) \leftrightarrow \frac{1}{s+5}$$

Taking Laplace for both side of the differential Eqn.

$$\Rightarrow [s^2 Y(s) - s + 1] + 4[sY(s) - 1] + 3Y(s) = \frac{1}{s+5}$$

solving for  $Y(s) \Rightarrow Y(s) = \frac{1}{(s+5)(s^2+4s+3)} + \frac{(s+3)}{(s^2+4s+3)}$

$$Y(s) = \frac{1}{(s+5)(s+3)(s+1)} + \frac{(s+3)}{(s+1)(s+3)}$$

$$= \frac{A_1}{(s+5)} + \frac{A_2}{(s+3)} + \frac{A_3}{(s+1)} + \frac{1}{(s+3)}$$

Continue  $\Rightarrow$

$$A_1 = \left. \frac{1}{(s+3)(s+1)} \right|_{s=-5} = \frac{1}{8}$$

$$A_2 = \left. \frac{1}{(s+5)(s+1)} \right|_{s=-3} = -\frac{1}{4}$$

$$A_3 = \left. \frac{1}{(s+5)(s+3)} \right|_{s=-1} = \frac{1}{8}$$

$$Y(s) = \frac{1/8}{(s+5)} - \frac{1/4}{(s+3)} + \frac{1/8}{(s+1)} + \frac{1}{(s+1)}$$

$$= \frac{1/8}{(s+5)} - \frac{1/4}{(s+3)} + \frac{9/8}{(s+1)}$$

$$\Rightarrow y(t) = \left[ \frac{1}{8} e^{-5t} - \frac{1}{4} e^{-3t} + \frac{9}{8} e^{-t} \right] u(t)$$