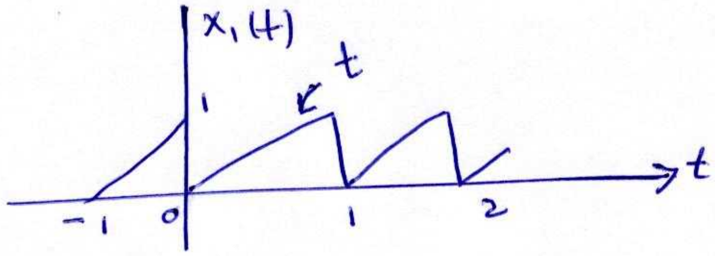
EE 207 _ W09 (082)

Extra Problem #2



$T_0 = 1$ sec

$$a_0 = \frac{1}{T_0}\int_{T_0} x_1(t)\,dt = \cancel{\tfrac{1}{4}}\ \frac{1}{1}\left[\left(\tfrac{1}{2}\right)(1)\right] = \boxed{\tfrac{1}{2}}$$

$$a_n = \frac{2}{T_0}\int_{T_0} x_1(t)\cos\frac{2\pi n t}{1}\,dt$$

$$= \frac{2}{1}\int_0^1 t\cos 2\pi n t\,dt$$

Integrating by Parts $\Rightarrow$ $u = t$ $\quad dv = \cos 2\pi n t\,dt$

$du = dt \quad v = \dfrac{\sin 2\pi n t}{2\pi n}$

$$\Rightarrow a_n = 2t\,\frac{\sin 2\pi n t}{2\pi n}\Big|_0^1 - 2\int_0^1 \frac{\sin 2\pi n t}{2\pi n}\,dt$$

$$= \frac{2}{2\pi n}\left[(1)\sin 2\pi n - (0)\sin(0)\right] + \frac{2}{(2\pi n)^2}\left[\cos(2\pi n t)\right]_0^1$$

$$= \frac{2}{2\pi n}\left[0 - 0\right] + \frac{2}{(2\pi n)^2}\left[\cos 2\pi n - \cos(0)\right]$$

$$= 0 + \frac{2}{(2\pi n)^2}\left[1 - 1\right] = \boxed{0}$$

$$b_n = \frac{2}{T_0} \int_{T_0} x_1(t) \sin(2\pi n t)\, dt$$

$$= \frac{2}{1} \int_0^1 t \sin(2\pi n t)\, dt$$

Integrating by Parts $\quad u = t \quad dv = \sin(2\pi n t)\, dt$

$du = dt \quad v = -\frac{\cos(2\pi n t)}{2\pi n}$

$$\Rightarrow b_n = \frac{-2}{2\pi n}\left[t \cos 2\pi n t\right]_0^1 + \frac{2}{2\pi n}\int_0^1 \cos(2\pi n t)\, dt$$

$$= -\frac{1}{n\pi}\left[(1)\cos 2\pi n - (0)\cos(0)\right] + \frac{2}{(2\pi n)^2} \sin(2\pi n t)\Big|_0^1$$

$$= -\frac{1}{n\pi} + \frac{2}{(2\pi n)^2}\left[\sin 2\pi n - 0\right]$$

$$= \boxed{\frac{-1}{n\pi}}$$

$$\Rightarrow x_1(t) = \frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{1}{n} \sin 2\pi n t$$

$$X_n = \frac{1}{T_0}\int_{T_0} x_1(t)\, e^{-j2\pi n t}\, dt$$

$$= \frac{1}{1}\int_0^1 t\, e^{-j2\pi n t}\, dt$$

Integrating by parts

$u = t \qquad dv = e^{-j2\pi n t}\, dt$

$du = dt \qquad v = -\frac{e^{-j2\pi n t}}{j2\pi n}$

$$\Rightarrow X_n = -\left.\frac{t e^{-j2\pi n t}}{j2\pi n}\right|_0^1 + \frac{1}{j2\pi n}\int_0^1 e^{-j2\pi n t}\, dt$$

$$= -\frac{1}{j2\pi n}\left[e^{-j2\pi n} - 0\right] - \left.\frac{1}{(j2\pi n)^2}\, e^{-j2\pi n t}\right|_0^1$$

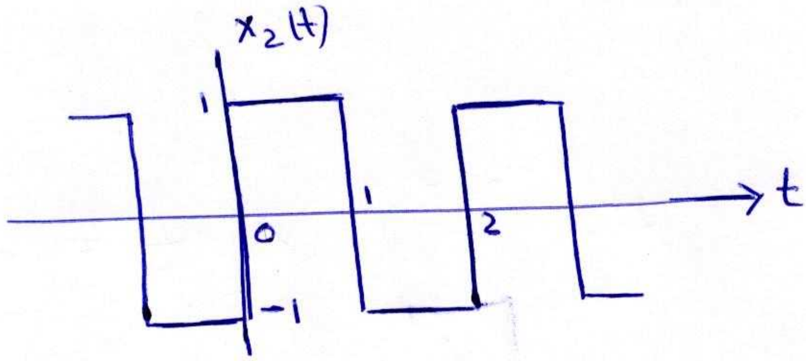
$$= -\frac{1}{j2\pi n}\left[1 - 0\right] - \frac{1}{(j2\pi n)^2}\left[e^{-j2\pi n} - e^{0}\right]$$

$$= -\frac{1}{j2\pi n} - \frac{1}{(j2\pi n)^2}\left[1 - 1\right]$$

$$= \boxed{-\frac{1}{j2\pi n}}$$

Note $X_n = \frac{1}{2}a_n - \frac{1}{2}j b_n$

$= 0 - \frac{1}{2}j\left(-\frac{1}{n\pi}\right)$



$T_0 = 2$ sec

$x_2(t)$ is odd function $\Rightarrow$ $a_0 = 0$ $\quad a_n = 0$

$$b_n = \frac{2}{T_0}\int_{T_0} x_2(t) \sin \frac{2\pi n t}{2}$$

$$= \frac{2}{2}\left[\int_0^1 (1)\sin(\pi n t)\,dt + \int_1^2 (-1)\sin(\pi n t)\,dt\right]$$

$$= -\frac{\cos \pi n t}{\pi n}\Bigg|_0^1 + \frac{\cos(\pi n t)}{\pi n}\Bigg|_1^2$$

$$= -\frac{\cos n\pi(1) - \cos(0)}{\pi n} + \frac{1}{\pi n}\left[\cos(\pi n(2)) - \cos(\pi n(1))\right]$$

n-odd

$\cos(n\pi) = -1 \qquad \cos(2\pi n) = 1$

$$b_n = \frac{2}{\pi n} + \frac{1}{\pi n}(1 - (-1)) = \frac{4}{n\pi}$$

n-even

$\cos(n\pi) = 1 \qquad \cos(2n\pi) = 1 \quad \Rightarrow \quad b_n = 0$

$$\Rightarrow \quad b_n = \boxed{\frac{4}{n\pi}} \quad n\text{-odd}$$

$$X_n = \frac{1}{T_0}\int_{T_0} x_2(t)\, e^{-j\frac{2\pi n t}{2}}\, dt$$

$$= \frac{1}{2}\left[\int_0^1 (1)\, e^{-j\pi n t}\, dt + \int_1^2 (-1)\, e^{-j\pi n t}\, dt\right]$$

$$= \frac{1}{2}\left[\left.\frac{e^{-j\pi n t}}{-j\pi n}\right|_0^1 - \left.\frac{e^{-j\pi n t}}{-j\pi n}\right|_1^2\right]$$

$$= \frac{-1}{j2\pi n}\left[\left(e^{-j\pi n} - e^{0}\right) - \left(e^{-j2\pi n} - e^{-j\pi n}\right)\right]$$

Since $e^{-j2\pi n} = 1$ for all n (odd and even)

~~$e^{-j\pi n}$~~

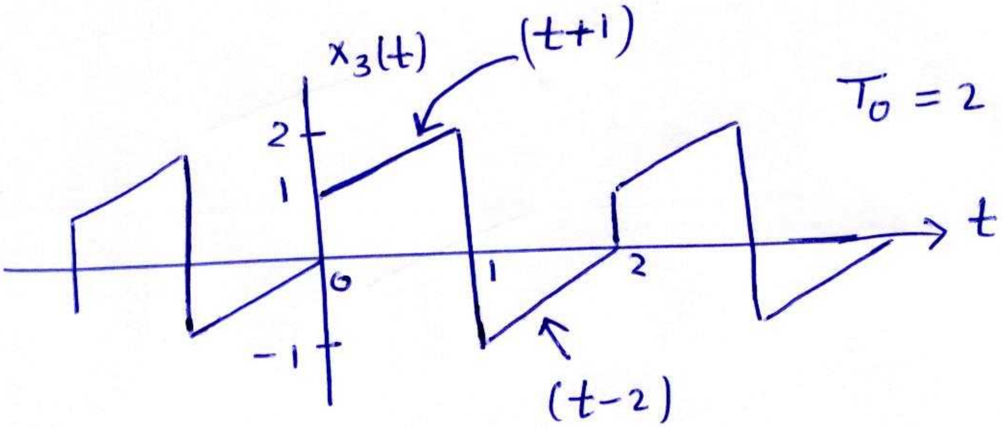
$$e^{-j\pi n} = \begin{cases} 1 & \text{for } n \text{ even} \\ -1 & \text{for } n \text{ odd} \end{cases}$$

$\Rightarrow$ n - odd $\quad X_n = -\frac{1}{j2\pi n}\left[\left((-1)-1\right) - \left(1-(-1)\right)\right] = \frac{2}{j\pi n}$

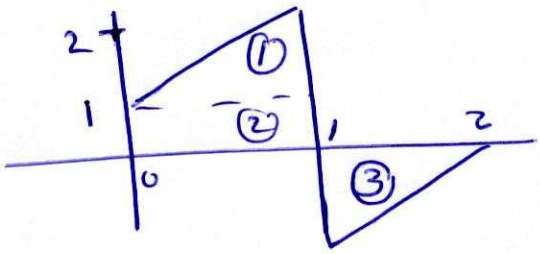
n - even $\quad X_n = 0$

$$\Rightarrow X_n = \begin{cases} 0 & n \text{ even} \\ \frac{2}{j\pi n} & n \text{ odd} \end{cases}$$

$T_0 = 2$



$$a_0 = \frac{1}{T_0}\int_{T_0} x_3(t)\,dt = \frac{1}{T_0}\,[\text{Area under one period}]$$



Area = 1 + 2 + 3

Since 1 = − 3

$\Rightarrow$ Area = 2 = (1)(1) = 1

$$\Rightarrow a_0 = \frac{1}{2}[1] = \boxed{\frac{1}{2}}$$

<u>Note</u> this is easier than $\displaystyle\int_0^1 (t+1)\,dt + \int_1^2 (t-2)\,dt$

$= 1$

$$a_n = \frac{2}{2}\left[\int_0^1 (t+1)\cos n\pi t\,dt + \int_1^2 (t-2)\cos n\pi t\,dt\right]$$

$$= \underbrace{\int_0^1 t\cos n\pi t\,dt}_{1} + \underbrace{\int_0^1 \cos n\pi t\,dt}_{2} + \underbrace{\int_1^2 t\cos n\pi t\,dt}_{3}$$

$$- 2\underbrace{\int_1^2 \cos n\pi t\,dt}_{4}$$

$$1 \Rightarrow \int_0^1 t\cos n\pi t\,dt = \begin{cases} \frac{-2}{(\pi n)^2} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

(integrating by parts)

$$2 \Rightarrow \int_0^1 \cos n\pi t\,dt = 0$$

$$3 \Rightarrow \int_1^2 t\cos n\pi t\,dt = \begin{cases} \frac{+2}{(\pi n)^2} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

$$4 \Rightarrow 2\int_1^2 \cos n\pi t\,dt = 0$$

$$\Rightarrow a_n = 1 + 2 + 3 + 4 = \begin{cases} 0 & \text{odd} \\ 0 & \text{even} \end{cases} = \boxed{0}$$

$$b_n = \frac{2}{T_0}\int_{T_0} x_3(t)\, \sin \frac{2n\pi t}{T_0}\, dt$$

$$= \frac{2}{2}\left[\int_0^1 (t+1)\sin n\pi t\, dt + \int_1^2 (t-2)\sin n\pi t\, dt\right]$$

$$= \underbrace{\int_0^1 t \sin n\pi t\, dt}_{①} + \underbrace{\int_0^1 \sin n\pi t\, dt}_{②} + \underbrace{\int_1^2 t \sin n\pi t\, dt}_{③}$$

$$- 2\underbrace{\int_1^2 \sin n\pi t\, dt}_{④}$$

$$① \Rightarrow \int_0^1 t \sin n\pi t\, dt = \begin{cases} \frac{1}{\pi n} & n\text{-odd} \\ -\frac{1}{n\pi} & n\text{-even} \end{cases}$$

$$② \Rightarrow \int_0^1 \sin n\pi t\, dt = \begin{cases} \frac{2}{n\pi} & n\text{-odd} \\ 0 & n\text{-even} \end{cases}$$

$$③ \Rightarrow \int_1^2 t \sin n\pi t\, dt = \begin{cases} -\frac{3}{\pi n} & n\text{-odd} \\ -\frac{1}{\pi n} & n\text{-even} \end{cases}$$

$$④ \Rightarrow -2\int_1^2 \sin n\pi t\, dt = \begin{cases} \frac{4}{n\pi} & n\text{-odd} \\ 0 & n\text{-even} \end{cases}$$

$$\Rightarrow b_n = ① + ② + ③ + ④$$

$$b_n = \begin{cases} \frac{4}{n\pi} & n\text{-odd} \\ -\frac{2}{n\pi} & n\text{-even} \end{cases}$$

$$X_n = \frac{1}{T_0}\int_{T_0} x_3(t)\, e^{-\frac{j2\pi n t}{T_0}}\, dt$$

$$= \frac{1}{2}\left[\int_0^1 (t+1)\, e^{-\frac{j2\pi n t}{2}}\, dt + \int_1^2 (t-2)\, e^{-\frac{j2\pi n t}{2}}\, dt\right]$$

$$= \frac{1}{2}\int_0^1 t e^{-j\pi n t}\, dt + \frac{1}{2}\int_0^1 e^{-j\pi n t}\, dt + \frac{1}{2}\int_1^2 t e^{-j\pi n t}\, dt - \frac{2}{2}\int_1^2 e^{-j\pi n t}\, dt$$

$$= \begin{cases} -\frac{1}{jn\pi} & n\text{-even} \\ \frac{2}{jn\pi} & n\text{-odd} \end{cases}$$