Optimal Placement of Wireless Base-Stations Based on 2-D Convolution

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***Abstract* – This work introduces a new technique for solving the optimal placement of wireless base-stations. The technique allows the use of any arbitrary coverage demand patterns as well as any antenna propagation patterns. The technique is based on numerical computations in which the 2-D convolution is a core operation. Simulation results show the effectiveness of such approach in solving the placement problem.**

***Index Terms* – Optimal Placement, Wireless Base Stations, Efficient algorithm, Convolution.**

I. INTRODUCTION

In wireless network design, the placement of wireless base-stations (BS) is a critical and challenging problem. The placement problem has conflicting requirements such as cost and Quality of Service (QoS). Increasing the number of BSs usually improves the service but has a direct impact on the cost of the network. Therefore, one of the main objectives of the placement problem is minimize the number of BSs such that a certain level of QoS is achieved.

In the literature, there has been many attempts to automate the process of base-stations placement. The attempts try to find a simplified optimization model for the problem and then solve it using some common optimization tools. In [1], the problem is modelled as a nonlinear program and then solved using three nonlinear optimization algorithms. The work in [2] formulated the placement problem as large-scale combinatorial optimization model and the simulated-annealing approach is then used to solve it. Similar model was developed in [3]. The model in this work was solved using generic algorithms resulting in sub-optimal solutions. In [4], a variant of the Simplex method was used to solve the placement problem where the objective here is to maximize the percentage coverage. The efficiency of this solution method to tackle the placement problems was highlighted.

In this work, we consider a new approach for obtaining a solution for the placement problem. This approach mainly finds the minimum number of base stations as well as their locations such that the signal power all over the area under consideration is above a certain threshold. The approach is based on the computation of the contribution of each point inside the geographical grid in case it is chosen as potential candidate for BS location. The 2-D convolution is used to compute the contributions of these points based on which the location of the base-stations is determined. The advantage of this approach over conventional methods is that it allows the use of any arbitrary coverage demand and antenna propagation patterns. This allows the application of this approach to wide scope of placement configurations.

II. PROBLEM STATEMENT

Let N be the number of base stations placed inside a rectangular geographical region. The objective of the placement problem is to minimize N so that the signal power inside the whole area is at least α. In other words,

$$p(x, y) \geq \alpha \qquad \forall x, y \tag{1}$$

where $p(x, y)$ is the effective signal power at location (x, y) while α is the minimum signal power. The signal power at location (x, y) is taken as the maximum power received from any of the serving base stations. In other words, the mobile stations at location (x, y) connects to base-station with strongest signal at that location. To solve the placement problem, it is first discretized into finite number of grid points. The number of divisions depends on the required resolution. In matrix format, the problem can be written as

$$\min \quad N \tag{2}$$

subject to

$$P \geq \alpha \tag{3}$$

where P is the power matrix all over the rectangular area and α is a constant power threshold. This constraint states that all the elements of the matrix P should be greater than the power threshold α. The power matrix can be represented as

$$P = \max_{n} \ \{A * X_n\} - F \tag{4}$$

where A is the antenna propagation matrix and F is the coverage pattern matrix. The matrix A is determined by the shape of the antenna propagation pattern. Fig. 1 shows four different examples of antenna patterns that can be modelled by the proposed scheme. The demand matrix F determines all coverage priority and environment patterns including terrain and fading characteristics. The location matrix X_n is all zeros except at the location of the BS n, denoted by (u_n, v_n), where the value is set to "1". In other words,

$$X_n(i,j) = \begin{cases} 1 & \text{at} \quad (u_n, v_n) \\ 0 & \text{elsewhere.} \end{cases} \tag{5}$$

Therefore, the term $A * X_n$ appeared in (4) determines the power contribution of the base station n located at the point (u_n, v_n). This representation of the placement problem in matrix format helps in borrowing useful tools from matrix theory in order to find the optimal solution as will be shown in the next section.

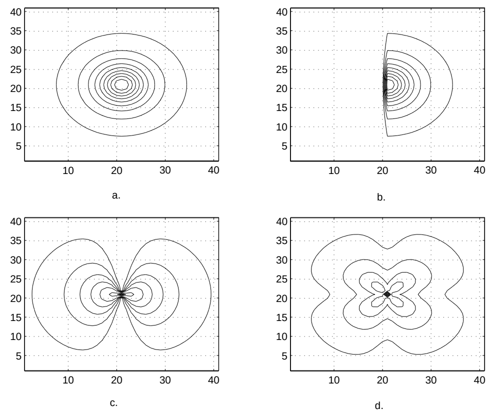

Figure 1. *Examples of four different antenna propagation patterns.*

III. Solution of the Placement Problem

The optimization problem (2-5) is solved using the following approach. The designer first provides fixed propagation and demand patterns A, and F. Then, the algorithm will compute the *power contribution* of every point on the grid to its neighboring points in case it is chosen as a BS location. This, off course, takes into account the given demand pattern. Then the point that delivers the *minimum power contribution* is chosen as the new base-station location. After placing each station, the power pattern P is updated and process repeats to choose other base-stations. The algorithm stops when the minimum power inside the covered area is higher than the threshold α.

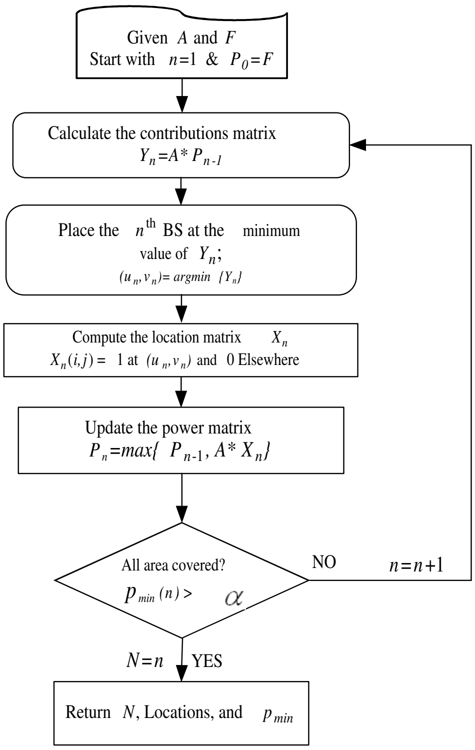

Figure 2. *The proposed solution algorithm.*

A flow chart of the proposed algorithm is shown in Fig. 2. The initial step is to set the signal power matrix P_0 to be equal to the demand pattern matrix F. Then, to determine the contribution of each point on the grid to the signal power in case it is chosen, the propagation pattern A is convolved with the current power pattern P_n

$$Y_n = A * P_n \tag{6}$$

where the start $*$ indicates the 2-dimensional convolution. The role of the convolution here is as follows. For each point on P_n, the matrix A is centered at that point and dot-multiplied with the intersecting sector of P_n. The multiplication values are then summed up and the answer is stored at the corresponding point in Y_n. This convolution process is repeated for all other points in P_n. The resulting matrix Y_n will then represent the power contribution of each point on the grid if it is chosen as the next base station location. The coordinates (u_n, v_n) that correspond to the minimum value of the matrix Y_n are chosen as the location of the new base-station.

Once a new base-station location is chosen, a new matrix X_n is constructed with all zero elements except at the

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chosen location (u_n, v_n) where it is set to "1". The matrix X_n now defines the coordinates of the new base-station.

After choosing the location of the base station, its contribution is then computed by simply convolving the propagation matrix A with the matrix X_n

$$C_n = A * X_n \tag{7}$$

The updated power pattern is computed by taking the maximum of the current power pattern P_n and the power contribution by the new base-station C_n. In other words, the updated power pattern is

$$P_{n+1} = \max \ \{P_n, C_n\} \tag{8}$$

Notice that this expression is a recursive version of (4). After choosing the first base-station, equations (6-8) are repeated to choose the other base-stations. The algorithm terminates when the constraint (1) is satisfied (or the minimum power p_{min} exceeds the threshold α). The algorithm then returns the number of stations N, their locations, and the minimum power value p_{min}.

III-1. Penalizing Boundaries of the Grid

Based on the minimum contribution requirement, the algorithm of Fig. 2 tends to assign BSs at the boundaries so that the base-stations will be furthest apart from each other. This will be at the cost of increasing the required number of BSs. This problem can be resolved by augmenting one frame of values around the demand matrix F as shown in Fig. 3. The algorithm will then automatically find the optimum frame value that minimized the total number of base stations. The optimal frame value is usually a positive number that depends on the size of the matrices A and F and their values. In this study, the algorithm implements a simple line-search to find the optimum frame value. If two frame values give the same number of BSs, the algorithm will choose the one with the higher minimum power p_{min}. In this way, not only the number of stations will be minimized but also the minimum power will be maximized reflecting an improved over-all coverage.

III-2. Computations Complexity

By looking at the proposed algorithm of Fig. 2, we notice that the main computationally expensive operation is the convolution operation represented by (6). There are many ways that can be adopted to substantially reduce the complexity of this operation. The first one is to notice that the matrix P usually stars with structure that is full with zeros. Therefore, the sparsity of this matrix can be exploited to reduce the complexity (there is no need to compute the convolution at the zero values of the matrices). Furthermore, there is no need to compute the convolution at those points already covered by previously assigned base-stations. In this way, the areas where the convolution need to be computed can substantially decrease as new base-stations are assigned. Another way of reducing the complex operations is to import the efficient techniques discussed in the literatures to efficiently compute the convolution, see for example [5] and [6].

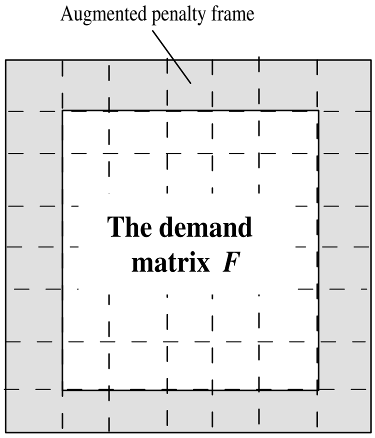

Figure 3. *Augmenting a penalty frame values around the demand matrix to avoid placement of base-stations at the boundaries.*

Notice that the other convolution operation appeared in the algorithm and is represented by (7) is not expensive to compute. This is because the matrix X has all its elements equal to zero except at the location (u_n, v_n) where it equals to unity. Therefore, this operation can be conducted simply by shifting the matrix A by (u_n, v_n).

IV. SIMULATION

A Matlab program was built to implement the algorithm of Fig. 2. The program uses the propagation and demand matrices A and F to compute the minimum number of BSs. The program returns also the location of these base stations as well as the percentage coverage of each BS.

To test the algorithm, it is first applied to a simple problem of covering a straight-line road. The length of the road is 10KM. An omni-directional antenna with a coverage diameter of 2KM is assumed. In this simple problem, other signal attenuations and interferences are not considered. The solution of this problem is straight-forward, namely, five base-stations should be placed uniformly along the road. The algorithm returned the placement results shown in Fig. 4. As expected, the algorithm obtained the correct number of base stations. Moreover, the BSs were aligned uniformly along the road as expected. The BSs in-

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dices at the centers of the cells indicate the order by which these stations were assigned. The percentage coverage of the base-stations is also returned by the algorithm and is shown in Fig. 5. Each of the BSs covered an equal amount of 20% out of the total demand space.

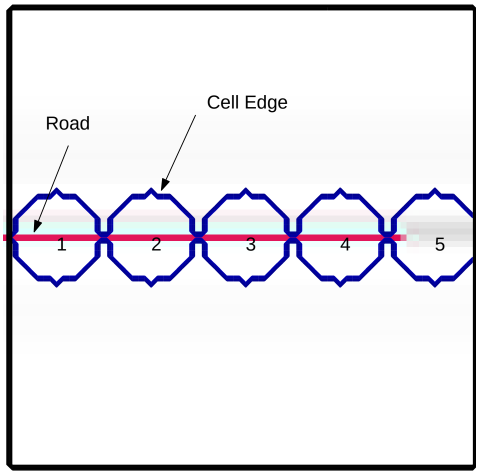

Figure 4. *Result of the optimal placement problem in the first example.*

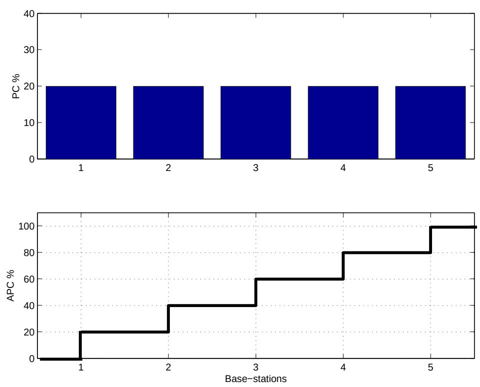

Figure 5. *Percentage coverage (PC) and accumulative percentage coverage (APC) of the base-stations in the first example.*

In another example, we consider a case where the road contains some curvatures. The curves were chosen, in purpose, to have a radius close to that of the omni-directional antenna. In this case, the algorithm returned the placement results shown in Fig. 6. As expected, the algorithm placed the first two stations at the centers of the two curvatures. Consequently, the percentage coverage by these two stations is maximum. The algorithm then placed more base stations uniformly to cover the remaining areas of the road. Notice that the last BS (BS $\#$ 6) is pushed inside the rectangular region since that is better than wasting the coverage power outside the demand region. This is achieved automatically by the algorithm through choosing proper frame penalty as discussed in section III-1. The percentage coverage for this example is shown in Fig. 7 where the first two BSs covered about 57% out of the total road path.

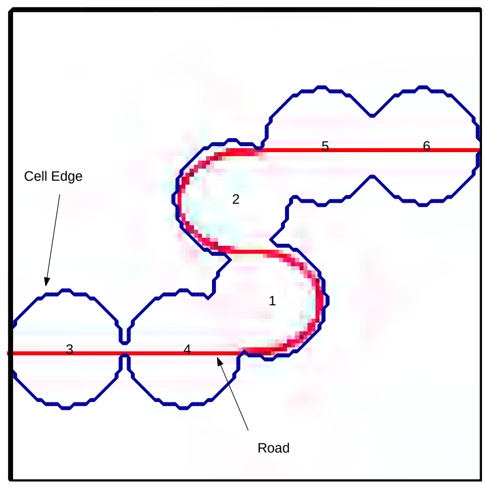

Figure 6. *Result of the optimal placement problem in the second example.*

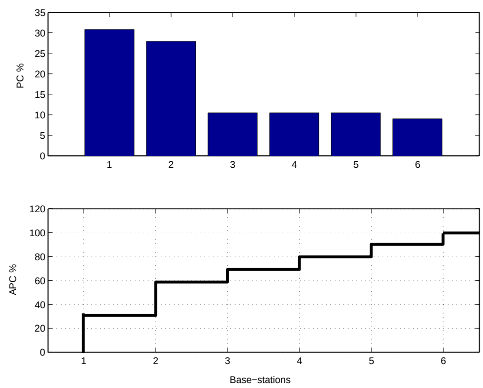

Figure 7. *Percentage coverage (PC) and accumulative percentage coverage (APC) of the base-stations in the second example.*

V. Conclusion

This paper introduces a new approach for placing wireless base stations. This approach is based on the convolution theory. The approach provides extended flexibility in choosing any arbitrary coverage demand and propagation patterns. Simulations were used to verify the new approach by applying it to known problems. Finally, the approach can be made more efficient by utilizing the advances made in the computation of the 2-D convolution.

VI. Acknowledgment

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