IMPACT OF INDUCTION MACHINE MODEL IN FAULT STUDIES

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Abstract

A hybrid second-order model of an induction motor is suggested and introduced in the common balanced faults studies. The suggested model is used in conjunction with a Thevenin equivalent circuit to trace the power system voltage patterns during fault and post-fault conditions. The accuracy of the proposed model is checked by comparing its results with the results of a detailled modelling of a two-bus power system netwok. Satisfactory results are reached.

Keywords: Faults Studies, Induction Machine Modelling, Voltage sag, Power Flow**.**

1- Introduction

It is a common trend that in formulating the necessary power flow equations for any power network during faults studies, the loads are usually either neglected or treated as static loads [1], [2]. In the case of static loads, constant impedances or constant powers are used to represent the loads. The solutions of the formulated power flow equations result in the prediction of voltage dips at the different buses of the examined power network. The voltage dips have usually a rectangular waveshape [3], that is characterized by sharp edges. The sharp edges correspond to the instant of fault occurrence and to the instant of fault clearance. Unfortunately in practice, the experience indicates that the voltage dips neither decrease sharply nor increase abruptly when a power is subjected to any fault condition [3]. One of the reasons that may lie behind such an experienced fact is the ignorance of the load dyamics when formulating the power flow equations. Dynamic loads models were not problably of much importance a few decades ago but they are becoming of great significance nowadays. The importance can be justified by the tremendeous application of sensitive electrical equipments, and in particular sensitive loads, that are quite vulnerable to any unexpected distortion in the supply voltage. In addition to the sensitivity of the loads, dynamic loads, precisely induction motors, are found to be leading in different applications when compared to the case of static loads applications. As a documented fact, induction motors are already constituing 60% of power system loads [4]. Therefore, without any doubt that the accurate model of load in any power system analysis study, like fault study, helps in the right prediction of voltage and provides hint about the possibility of loads stalling.

This paper derives a second order model for an induction motor, then shows later how can the dynamic model be included in fault studies. Thevenin theorem along the developed machine model are used to predict the induction machine bus terminal voltage. In the Thevenin equivalent circuit, the Thevenin voltage is found from the regular power flow solutions whereas the Thevenin impedance is found from the impedance matrix, that usually calculated in fault studies.
Results of the Thevenin equivalent circuit are compared with the results of a full modelling of typical two-bus power system having an induction motor in one of its buses.

2- System Studied

Figure 1 shows a two-bus power network containing a static load and a dynamic load (i.e induction motor). The system symbols and parameter values are provided in Appendices A & B. The two loads were chosen to have the same power rating during normal operating conditions. To derive a dynamic model for such a system, the following assumptions are made:

- The system is balanced and the fault will be a bolted three phase symmetrical fault. The fault occurs at bus 1 at a certain time then it is cleared later.

• The induction machine is a squirrel cage type machine having a uniform air gap.
• There are linear relationships betweem flux linkages and their respective currents.
• Stator transients of the flux linkages are neglected.

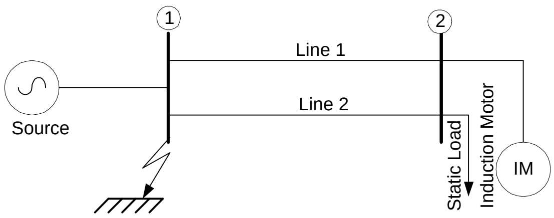


Figure 1: Two-bus Power Network

Based on the last assumption, the derivation of the induction machine model will be started from a third-order model. The third order model is reported [4] to be an accurate model when looking for the behaviour of the induction machine that is the subject to a large excursion in the terminal voltage. The third order model as well as the different analytical expressions for the buses voltage, line currents, flux linkages are based on the background work documented in reference [5].

I- System Model:

Following the analysis performed in appendix C, the induction machine dynamic model can be described by the following differential equations:

$$\frac{di_{rq}}{dt} = \left[i_{rq}(-R_r) + i_{rd}\left[-s(X_r + X_m)\right]\right]\frac{(X_s + X_m)w_b}{X_sX_r + X_m(X_s + X_r)} + \left[i_{md}(-sX_m)\right]\frac{(X_s + X_m)w_b}{X_sX_r + X_m(X_s + X_r)} \tag{1}$$

$$\frac{di_{rd}}{dt} = \left[i_{rd}(-R_r) + i_{rq}\left[s(X_r + X_m)\right]\right]\frac{(X_s + X_m)w_b}{X_sX_r + X_m(X_s + X_r)} + \left[i_{mq}(sX_m)\right]\frac{(X_s + X_m)w_b}{X_sX_r + X_m(X_s + X_r)} \tag{2}$$

$$\frac{dw_r}{dt} = \frac{1}{J\left(\frac{2}{p}\right)}(T_e - T_L) \tag{3}$$

where

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\frac{X_m}{w_b}(i_{mq}i_{rd} - i_{rq}i_{md})$$

Defining new transformation variables in the form of:

$$\begin{aligned} i_s &= i_{mq} - ji_{md} \\ i_r &= i_{rq} - ji_{rd} \end{aligned} \tag{4}$$

Actually, the new variables represent the phasor quantities of the stator and rotor currents respectively.

Equations (1) and (2) can be merged to give one complex differential equation of the form:

$$\frac{di_r}{dt} = \left[-\left[R_r + js(X_r + X_m)\right]i_r\right]\frac{(X_s + X_m)w_b}{X_sX_r + X_m(X_s + X_r)} + \left[-j(sX_m)i_s\right]\frac{(X_s + X_m)w_b}{X_sX_r + X_m(X_s + X_r)} \tag{5}$$

The expression of the electromagnetic torque in function of the new currents (i_s and i_r) can be checked to be in the form of :

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\frac{X_m}{w_b}\operatorname{Im}ag(i_s i_r^{\ *}) \tag{6}$$

Equations (5) and (3) will therefore constitue a hybrid <u>second order model </u>for the induction machine.

In order to avoid all d-axis and q-axis quantities in the system model, the algebraic expressions of equations (A-14)-(A-16) are also merged and that is by considering the following transformations:

$$\begin{aligned} v_{os} &= v_{oq} - jv_{od} \\ i_l &= i_{lq} - ji_{ld} \\ i_2 &= i_{2q} - ji_{2d} \end{aligned} \tag{7}$$

The results of the merging process end up with the following three algebraic equations:

$$v_{os} = \left[(R_0 + R_1) + j(X_0 + X_1)\right]i_s + \left[(R_l + R_0 + R_1)\right]i_l + \left[j(X_l + X_0 + X_1)\right]i_l - \left[R_1 + jX_1\right]i_2 \tag{8}$$

$$v_{os} = \left[R_0 + jX_0\right]i_s + \left[(R_0 + R_l)\right]i_l + \left[j(X_0 + X_l)\right]i_l + \left[R_2 + jX_2\right]i_2 \tag{9}$$

$$-jX_m i_r = \left[R_s + j(X_s + X_m)\right]i_s - \left[R_l + jX_l\right]i_l \tag{10}$$

Equations (8)-(10), (5), and (3) are all necessary equations to find the dynamic perfomance of the two bus system as well as the induction machine under healthy (non-faulty) operating conditions.

In the case of a bolted three phase symmetrical fault at bus 1, equations (5), (3), and (10) are still valid equations whereas equations (8) and (9) are altered by the following two equations:

$$0 = \left[R_1 + jX_1\right]i_s + \left[(R_l + R_1)\right]i_l + \left[j(X_l + X_1)\right]i_l - \left[R_1 + jX_1\right]i_2 \tag{11}$$

$$0 = \left[R_l + jX_l\right]i_l + \left[R_2 + jX_2\right]i_2 \tag{12}$$

The voltage at the induction machine bus during healthy conditions as well as fault conditions is calculated as:

$$v_{2s} = \left[R_s + j\left(X_s + X_m\right)\right] i_s + jX_m i_r \qquad (13)$$

II- System Model using Thevenin Theorem:

The derivation of the model for the previous two-bus system is not that lengthy and that because there are only two buses in the power network. But, in the case of a real power system, there is a large number of buses. Consequently, the derivation of all necessary algebraic equations of the system will certainly be a tedious task to undertake. One way of easing such a tedious task is to use Thevenin theorem approach. The approach consists of reducing any power system, when seen from the induction machine bus, to an equivalent Thevenin impedance ($Z_{th}=R_{th}+jX_{th}$) behind a Thevenin source voltage (v_{th}). Such reduced system is connected to the machine bus as shown in figure 2. The Thevenin source voltage is calculated from any common power flow algorithm. Of course, the induction machine model should not be considered in the power flow equations formulation. As to the Thevenin impedance, it is withdrawn from the diagonal elements of the impedance matrix of the system.

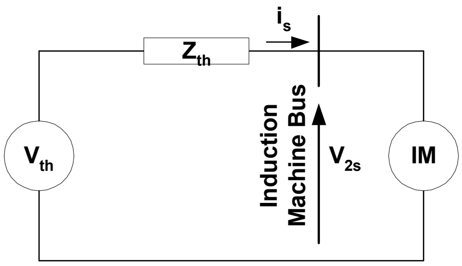


Figure 2: Thevenin Equivalent Circuit

The induction machine dynamic model in this case is:

$$\frac{di_r}{dt} = \left[-\left[R_r + js\left(X_r + X_m\right)\right] i_r\right] \frac{\left(X_s + X_{th} + X_m\right) w_b}{\left(X_s + X_{th}\right) X_r + \left(X_s + X_{th} + X_r\right) X_m}$$

$$+ \left[-\left(jsX_m\right) i_s\right] \frac{\left(X_s + X_{th} + X_m\right) w_b}{\left(X_s + X_{th}\right) X_r + \left(X_s + X_{th} + X_r\right) X_m} \qquad (24)$$

The stator current in the previous equation is calculated from the following algebraic expression:

$$i_s = \frac{V_{th} - jX_m i_r}{\left[R_s + R_{th} + j\left(X_s + X_{th} + X_m\right)\right]} \qquad (25)$$

The last equation (equation 25) is actually deduced from equation (13).

The voltage at the induction machine bus is simply calculated as:

$$V_{2s} = V_{th} - Z_{th} i_s \qquad (26)$$

III- System Model while treating the induction machine as a constant impedance static load:

When treating the induction motor as a constant impedance (R_{ind}+j X_{ind}), all equations of the two-bus power system become algebraic. Under healthy conditions, the following system of equations can be solved to find the currents i_s, i_l, i_2

$$\begin{bmatrix} v_{os} \\ v_{os} \\ 0 \end{bmatrix} = \begin{bmatrix} (R_0+R_1)+j(X_0+X_1) & (R_l+R_0+R_1)+j(X_l+X_0+X_1) & -(R_1+jX_1) \\ (R_0+jX_0) & (R_0+R_l)+j(X_0+X_l) & (R_2+jX_2) \\ (R_{ind}+jX_{ind}) & -(R_l+j*X_l) & 0 \end{bmatrix} X \begin{bmatrix} i_s \\ i_l \\ i_2 \end{bmatrix}$$

(27)

Whereas under faulty conditions, currents is, il, and i2 are all null.

The voltage at the induction bus is calculated as:

$$V_{2s} = (R_{ind} + jX_{ind}) i_s \qquad (28)$$

Resistance R_{ind} and reactance X_{ind} are calculated from the pre-fault steady state conditions of the induction motor.

3- Models Performance

The performance of the two bus system of figure 1 is assessed. Such assessment consists of:

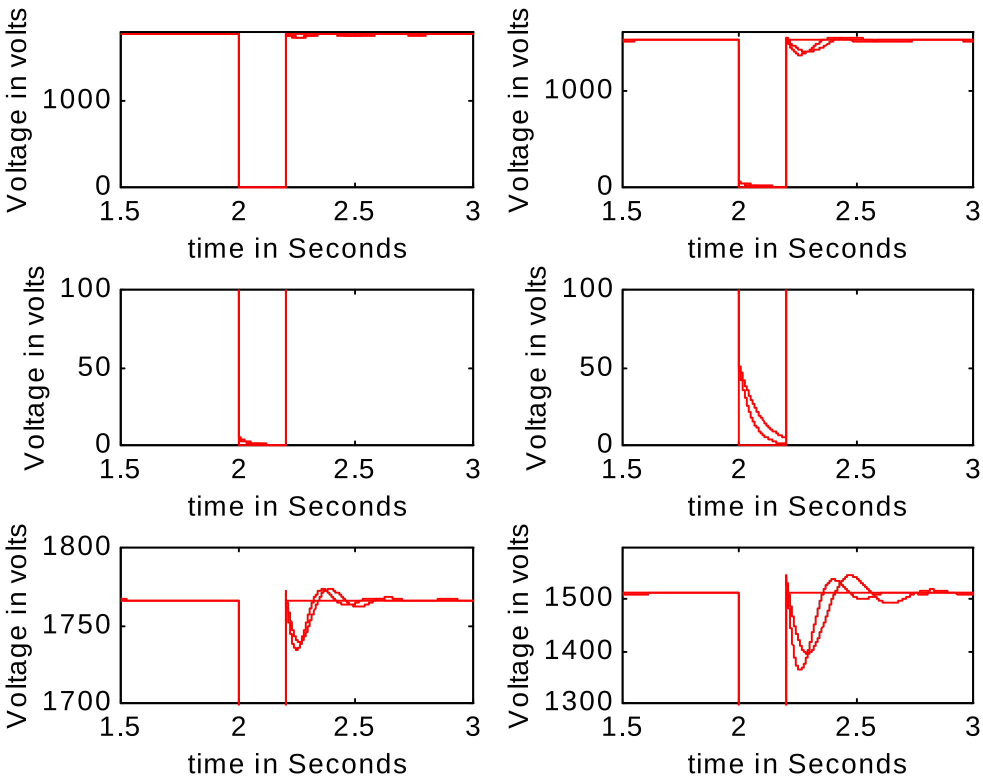
1- simulating the detailled model of the two bus system,

2- simulating the developed Thevenin model of the same two bus system,

3- simulating the two bus system while treating the induction machine as a constant impedance (i.e static load). The value of such impedance is obtained from pre-fault steady state conditions as has been mentioned earlier.

In all three simulations, the bolted three phase fault is assumed to be existing in bus 1 for a 200 ms time period.

The results of such simulations are shown in the left column of figure 3 (i.e. figure 3(a)). Such column represents the voltage at the induction bus. Note that in both columns of figure 3, the subfigures below the top subfigure are no more than a magnification of the voltage variation at the lower part and the upper part of top subfigure. In each subfigure of the coulmn, there are three curves. The solid curve, that is characterized by a rectangular voltage dip, represents the case of treating the induction machine load as a static load. The small dashed curve represents the case of the detailled model of the two bus bus system. The large dashed curve represents the case of the developed Thevenin model of the same two bus system.



(a) **(b)**

Figure 3: Induction Machine Terminal voltage **a)** when transmission lines impedance is 0.0558+j 0.0670 Ω **b)** when transmission lines impedance is 0.558+j 0.670 Ω

As it is observed, there is a quite reasonable agreement between the results of the developed Thevenin equivalent circuit and the ones of the detailled model of the two bus power network.

To make sure that the previous observation is always valid, the impedance of the lines connecting buses 1 and 2 are increased by a factor of 10 times. System simulations have been repeated and the right column of figure 3 represents the pattern of the voltage at the loads bus.

4- Conclusion

A method of including induction machine model in fault studies has been presented. The method predicts the voltage buses behaviour during pre-fault, fault, and post fault three phase symmetrical fault circumstances. The method consists of solve first the power flow algorithm, used in any conventional fault study, while ignoring the induction machine load. Such solution aims at predicting a Thevenin circuit at the induction machine bus. The Thevenin circuit is used in conjunction with a developed hybrid second-order model to estimate induction machine stator and rotor currents. The induction machine terminal voltage is calculated in the last stage. Method accuracy is checked with a detailled modelling of a two-bus power system. Reasonable agreement has been observed. Comparison of the method with the case of treating induction machine bus as a static load has been also considered. Quite clear deviations between the results of the suggested

method and the results of assuming the induction machine as a static load are encountered

5- References

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Appendices

Appendix A: List of Symbols

V_{so}:	Main source voltage	i_{1q}, i_{1d}:	q-axis and d-axis line 1 currents
R_o:	Lin 0 Resistance		
X_0:	Line 0 Reactance	i_{2q}, i_{2d}:	q-axis and d-axis line 2 currents
R_1:	Line 1 Resistance		
X_1:	Line 1 Reactance	I_{Lq}, I_{Ld}:	q-axis and d-axis load currents
R_2:	Line 2 Resistance		
X_2:	Line 2 Reactance	I_{mq}, I_{md}:	q-axis and d-axis induction machine stator currents
R_L:	Static Load Resistance		
X_L:	Static Load Reactance	I_{rq}, I_{rd}:	q-axis and d-axis induction machine rotor currents
R_s:	Induction machine stator resistance		
R_r:	Induction machine rotor resistance	ψ_{oq}, ψ_{od}:	q-axis and d-axis source terminal flux linkages
X_s:	Induction machine stator reactance		
X_r:	Induction machine rotor reactance	ψ_{1q}, ψ_{1d}: ψ_{2q}, ψ_{2d}:	q-axis and d-axis bus 1 flux linkages q-axis and d-axis bus 2 flux linkages
X_M:	Induction machine magnetizing reactance		
J:	Induction machine inertia constant	ψ_{mq}, ψ_{md}:	q-axis and d-axis induction machine stator flux linkages
T_L:	Applied load torque		
P	Number of machine poles	ψ_{2q}, ψ_{2d}: ψ_{Lq}, ψ_{Ld}:	q-axis and d-axis bus 2 flux linkages q-axis and d-axis static load flux linkages
S	Machine Slip		
v_{oq}, v_{od}:	q-axis and d-axis source terminal voltages	ω_b	Base angular velocity
v_{1q}, v_{1d}:	q-axis and d-axis bus 1 voltages	i_{1q}, i_{1d}:	q-axis and d-axis line 1 currents
v_{2q}, v_{2d}:	q-axis and d-axis bus 2 voltages	V_{th}, Z_{th}	Thevenin source voltage an equivalent impedance
i_{oq}, i_{od}:	q-axis and d-axis line 0 currents	R_{ind}, X_{ind}	Equivalent resistance and reactance of the induction machine when treating the latter as a static load.

Appendix B: Two bus Power System Data

Source	Voltage RMS value 2300/√3 V	R_0 = 0.0135 Ω	X_0 = 0.135 Ω
Lines	R_1 = 0.0558Ω	X_1 = 0.06699 Ω	
	R_2 = 0.0558Ω	X_2 = 0.06699 Ω	
Loads	Static Load R_l =2.4377 Ω X_1 =1.844 Ω		
	Induction machine R_s = 0.029 Ω R_r = 0.022 Ω X_s = 0.188 Ω X_r = 0.188 Ω X_m = 10.867 Ω J= 63.87 Kg-m^2 Rated Load Torque = 4000 N.m P= 4 number of machine poles		

Appendix C: Derivation of the Third-order Model Of the System of Figure 1

C-1: No-Fault condition:

For the system of figure 1, the analysis is started from the following equations:

- Voltage at bus 1:

$$v_{1q} = v_{0q} - \left[R_0 i_{0q} + y_{0d}\right]$$
$$v_{1d} = v_{0d} - \left[R_0 i_{0d} - y_{0q}\right] \qquad \text{(A-1)}$$

- Voltage at bus 2:

$$v_{2q} = v_{1q} - \left[R_1 i_{1q} + y_{1d}\right] = v_{1q} - \left[R_2 i_{2q} + y_{2d}\right]$$
$$v_{2d} = v_{1d} - \left[R_1 i_{1d} - y_{1q}\right] = v_{1d} - \left[R_2 i_{2d} - y_{2q}\right] \quad \text{(A-2)}$$

- Static Load at bus 2:

$$v_{2q} = \left[R_l i_{lq} + y_{ld}\right]$$
$$v_{2d} = \left[R_l i_{ld} - y_{lq}\right] \qquad \text{(A-3)}$$

- For the induction machine: :

$$v_{2q} = R_s i_{mq} + y_{md}$$
$$v_{2d} = R_s i_{md} - y_{mq} \qquad \text{(A-4)}$$

$$v_{rq} = 0 = R_r i_{rq} + s y_{rd} + \frac{1}{w_b}\frac{d}{dt} y_{rq}$$
$$v_{rd} = 0 = R_r i_{rd} - s y_{rq} + \frac{1}{w_b}\frac{d}{dt} y_{rd} \quad \text{(A-5)}$$

The relations between currents are as follows:

$$i_{0q} = i_{1q} + i_{2q} = i_{mq} + i_{lq}$$

$$i_{0d} = i_{1d} + i_{2d} = i_{md} + i_{ld} \qquad \text{(A-6)}$$

whereas the relationships between fluxes and currents are:

$$y_{0q} = i_{0q} X_o, \quad y_{0d} = i_{0d} X_o$$
$$y_{1q} = i_{1q} X_1, \quad y_{1d} = i_{1d} X_1$$
$$y_{2q} = i_{2q} X_2, \quad y_{2d} = i_{2d} X_2$$
$$y_{lq} = i_{lq} X_l, \quad y_{ld} = i_{ld} X_l \qquad \text{(A-7)}$$

$$y_{mq} = i_{mq}\left(X_s + X_m\right) + i_{rq} X_m$$
$$y_{md} = i_{md}\left(X_s + X_m\right) + i_{rd} X_m$$
$$y_{rq} = i_{rq}\left(X_r + X_m\right) + i_{mq} X_m$$
$$y_{rd} = i_{rd}\left(X_r + X_m\right) + i_{md} X_m$$

Bus 1 voltages (v_{1d} and v_{1q}) can be elemitated from systems of equations (A-1) and (A-2). That is,

$$v_{2q} = v_{0q} - \left[R_0 i_{0q} + y_{0d}\right] - \left[R_1 i_{1q} + y_{1d}\right]$$
$$v_{2d} = v_{0d} - \left[R_0 i_{0d} - y_{0q}\right] - \left[R_1 i_{1d} - y_{1q}\right] \qquad \text{(A-8)}$$

Also

$$v_{2q} = v_{0q} - \left[R_0 i_{0q} + y_{0d}\right] - \left[R_2 i_{2q} + y_{2d}\right]$$
$$v_{2d} = v_{0d} - \left[R_0 i_{0d} - y_{0q}\right] - \left[R_2 i_{2d} - y_{2q}\right] \qquad \text{(A-9)}$$

Using the flux linkages-currents relationships (A-7), and the source currents $i_{0q} = i_{mq}+i_{lq}$ and $i_{0d} = i_{md}+i_{ld}$ and expressing line 1 currents in function of other currents as:

$$i_{1q} = i_{mq} + i_{lq} - i_{2q} \quad \text{and} \quad i_{1d} = i_{md} + i_{ld} - i_{2d},$$

the previous two equations (A-8) and (A-9) will be:

$$v_{2q} = v_{0q} - i_{mq}(R_0 + R_1) - i_{lq}(R_0 + R_1) + i_{2q}(R_1)$$
$$- i_{md}(X_0 + X_1) - i_{ld}(X_0 + X_1) + i_{2d}(X_1) \qquad \text{(A- 10)}$$

$$v_{2d} = v_{od} - i_{md}(R_0 + R_1) - i_{ld}(R_0 + R_1) + i_{2d}(R_1)$$
$$+ i_{mq}(X_0 + X_1) + i_{lq}(X_0 + X_1) - i_{2q}(X_1)$$

$$v_{2q} = v_{0q} - i_{mq}(R_0) - i_{lq}(R_0) - i_{2q}(R_2) - i_{md}(X_0) - i_{ld}(X_0) - i_{2d}(X_2)$$
$$v_{2d} = v_{od} - i_{md}(R_0) - i_{ld}(R_0) - i_{2d}(R_2) + i_{mq}(X_0) + i_{lq}(X_0) + i_{2q}(X_2) \qquad \text{(A- 11)}$$

The static load voltages (equation A-3) in fucntion of the currents can be also expressed as:

$$v_{2q} = \left[R_l i_{lq} + X_l i_{ld}\right]$$
$$v_{2d} = \left[R_l i_{ld} - X_l i_{lq}\right] \qquad \text{(A- 12)}$$

Similarly, the induction machine stator voltages (equation A-4) can be written in the form:

$$v_{2q} = R_s i_{mq} + (X_s + X_m) i_{md} + X_m i_{rd}$$
$$v_{2d} = R_s i_{md} - (X_s + X_m) i_{mq} - X_m i_{rq} \qquad \text{(A- 13)}$$

Equating (A-10) with (A-12), then (A- 11) with (A- 12), the following system of equations is reached:

$$v_{oq} = i_{mq}(R_0 + R_1) + i_{lq}(R_l + R_0 + R_1) - i_{2q}(R_1) + i_{md}(X_0 + X_1)$$
$$+ i_{ld}(X_l + X_0 + X_1) - i_{2d}(X_1) \qquad \text{(A- 14)}$$

$$v_{od} = -i_{mq}(X_0 + X_1) - i_{lq}(X_l + X_0 + X_1) + i_{2q}(X_1)$$
$$+ i_{md}(R_0 + R_1) + i_{ld}(R_l + R_0 + R_1) - i_{2d}(R_1)$$

$$v_{oq} = i_{mq}(R_0) + i_{lq}(R_l + R_0) + i_{2q}(R_2) + i_{md}(X_0)$$
$$+ i_{ld}(X_l + X_0) + i_{2d}(X_2) \qquad \text{(A- 15)}$$

$$v_{od} = i_{mq}(-X_0) + i_{lq}(-X_l - X_0) + i_{2q}(-X_2)$$
$$+ i_{md}(R_0) + i_{ld}(R_l + R_0) + i_{2d}(R_2)$$

Also when equating induction machine voltages (A-13) with static load volatges (A-12) , one can get:

$$i_{mq}(R_s) + i_{lq}(-R_l) + i_{md}(X_s + X_m) + i_{ld}(-X_l) = i_{rd}(-X_m) \qquad \text{(A- 16)}$$

$$i_{mq}(-X_s - X_m) + i_{lq}(X_l) + i_{md}(R_s) + i_{ld}(-R_l) = i_{rq}(X_m)$$

The rotor equations (A-6), the following two differential equations can be easily derived:

$$\frac{di_{rq}}{dt} = \left[i_{rq}(-R_r) + i_{rd}\left[-s(X_r + X_m)\right] + i_{md}(-sX_m)\right]\frac{(X_s + X_m)w_b}{X_s X_r + X_m(X_s + X_r)} \qquad \text{(A- 17)}$$

$$\frac{di_{rd}}{dt} = \left[i_{rd}(-R_r) + i_{rq}\left[s(X_r + X_m)\right] + i_{mq}(sX_m)\right]\frac{(X_s + X_m)w_b}{X_s X_r + X_m(X_s + X_r)} \qquad \text{(A-18)}$$

The third differential equation of the motor needed is the equation of motion:

$$\frac{dw_r}{dt} = \frac{1}{J\left(\frac{2}{p}\right)}(T_e - T_L) \qquad \text{(A-19)}$$

Where, T_e is the electromagnetic torque and it carries the expression:

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\frac{X_m}{w_b}(i_{mq} i_{rd} - i_{rq} i_{md})$$

C-2: Under Fault condition:

The previous procedure can be reworked to find the system state equations when bus1 is grounded. The following analytical equations are obtained:

$$0 = i_{mq}(R_1) + i_{lq}(R_l + R_1) - i_{2q}(R_1) + i_{md}(X_1) + i_{ld}(X_l + X_1) - i_{2d}(X_1)$$
$$0 = -i_{mq}(X_1) - i_{lq}(X_l + X_1) + i_{2q}(X_1) + i_{md}(R_1) + i_{ld}(R_l + R_1) - i_{2d}(R_1) \qquad \text{(A- 20)}$$

$$0 = i_{lq}(R_l) + i_{2q}(R_2) + i_{ld}(X_l) + i_{2d}(X_2)$$
$$0 = i_{lq}(X_l) + i_{2q}(X_2) + i_{ld}(-R_l) - i_{2d}(R_2) \qquad \text{(A- 21)}$$

$$i_{mq}(R_s) + i_{lq}(-R_l) + i_{md}(X_s + X_m) + i_{ld}(-X_l) = i_{rd}(-X_m)$$
$$\text{(A- 22)}$$

The dynamic equations of the machine are the same equations mentionned earlier in (A-17)-(A-19).