MEMS AD/DA CONVERTERS

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ABSTRACT

MEMS Analog -to-digital and digital-to- analog converters are proposed using parallel plate electrostatic actuators under bias. Deformable plates supported by springs are used with bias applied voltage which determines the amount of static displacement needed for equilibrium condition. In analog-to-digital arrangement, eight different spring displacements are tapped off the spring corresponding to eight binary decoded voltages. At spring tapped connections, MOS switches are switched on connecting a digital high voltage level at these locations so that when a certain analog voltage is applied on the moving plate of the capacitor, the spring is displaced to one of these locations, enabling different binary voltages to all switches up to that level. The digital binary voltages are fed to an 8-3 priority encoder to obtain the digital value. In digital-to-analog arrangement, the input binary voltage is decoded to different spring locations which correspond to resistances making up a potentiometer circuit for the output analog voltage.

1. INTRODUCTION

The parallel plate capacitor with one movable plate supported by a mechanical spring is depicted in Figure (1), where the top plate is supported by a spring with the force constant being K_m. At rest the applied voltage, displacement and the mechanical restoring forces are zero. Gravity does not play an important role in the static analysis of micro devices because the mass of plates is generally very small and the gravitational force would not cause appreciable static displacement.

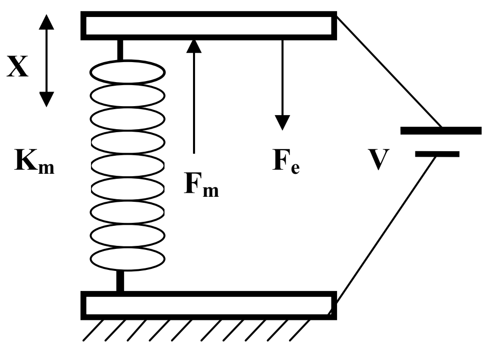

Figure (1) A coupled electromechanical model

When a voltage is applied an electrostatic force F_e will be developed with a magnitude of

$$F_e = \varepsilon A V^2/[2 d^2]$$

with the movable plate is at its starting position. This force tends to decrease the gap which gives rise to displacement and the mechanical restoring force. Under static equilibrium the mechanical restoring force has an equal magnitude but opposite direction as the electrostatic force. The magnitude of the electrostatic force is itself a function of the displacement. It's to be noted too that this electrostatic force affects the spring constant as well, due to the spring being softer due to this force. The spatial gradient of the electric force is defined as an electrical spring constant

$$K_e = \Delta F_e/\Delta d = CV^2/d^2$$

As seen the magnitude of electric spring constant changes with position d and the biasing voltage V. This is ignored for small displacements. Thus the effective spring constant is mechanical spring constant minus the electrical spring constant.

To derive the equilibrium displacement of a spring supported electrode plate under a bias voltage V, consider the resulting equilibrium displacement being x in the direction of increasing gap. With displacement x, the gap between the two electrode plates is d+x and thus the electrostatic force at equilibrium is

$$F_e = \varepsilon A V^2/[2 (X_o+X)^2] = C(X) V^2/[2 (X_o+X)]$$

Whereas the mechanical force is

$$F_m = -K_m X$$

By equating these two forces, and rearranging terms,

$$X = F_m/K_m = F_e/K_m = C(x) V^2/[2 (X_o+X) K_m]$$

The displacement at equilibrium can be calculated from the above quadratic equation as
This can be visualized graphically as shown in Figure (2). The horizontal axis represents space between the two plates, and the vertical axis is the mechanical or electrical

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force irrespective to their directions. The movable plate is displaced X_o from the rigid fixed plate at origin. Two curves representing both mechanical and electrical forces are plotted with electrode positions according to their quoted equations and their intersecting points correspond to the solutions of the above quadratic equation. It can be noted that more than one intersecting points exist, but only one is achieved in reality. The solution that is closest to the rest position is realized first and is generally the realistic solution.

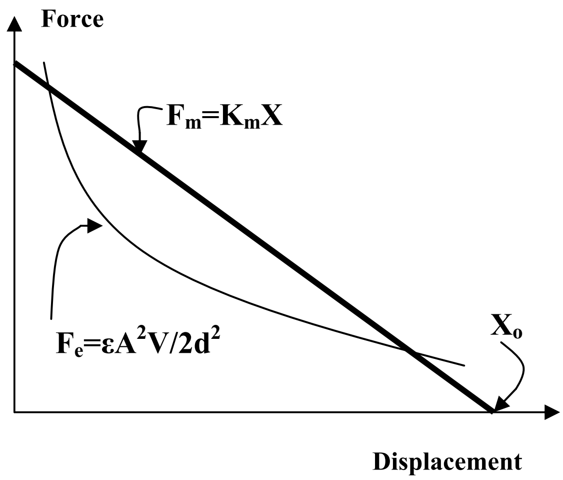


Figure (2) Electrical and mechanical forces as functions of spring displacement

The graphic solution can be used to track the equilibrium position as the bias voltage is increased. As the voltage increases, the family of curves corresponding to electrostatic force shifts upwards, shifting the x coordinates of the interception points further away from the rest position.

2. PULL-IN VOLTAGE

At a particular bias voltage the two curves intercept at one point tangentially as shown in Figure (3). At this interception point the electrostatic and mechanical force balance each other. At this point, the magnitude of electric and mechanical spring constants are equal. This is given by the gradient of the electrostatic force curve at the interception point, making the effective spring constant equal to zero, I.e. extremely soft. The bias voltage that invokes this condition is called the pull-in voltage V_p.

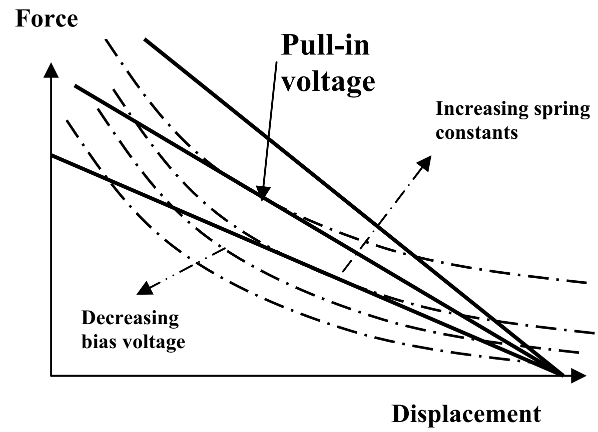


Figure (3) Balance of electrical and mechanical forces at different bias voltages and different spring constants

This pull-in voltage can be calculated as

$$V^2= -2k_mX(X+X_O)^2 /[\ \varepsilon\ A] = -2k_mX(X+X_O)/C$$

The value of x is negative when the spacing between the electrodes decreases.
If the bias voltage is increased further beyond V_p the two curves will not intercept and no equilibrium solution exists. In reality the electrostatic force continues to grow while the mechanical force is unable to catch up and match it. The two plates are thus pulled against each other rapidly until they contact, at which the mechanical force will finally balance the electric one. This condition is called pull in or snap in.

Now substituting the pull in voltage in the electric force constant equation $K_e= CV^2/d^2$ yields

$$K_e= CV^2/[X+X_o]^2 = -2K_mX/[X+X_o]$$

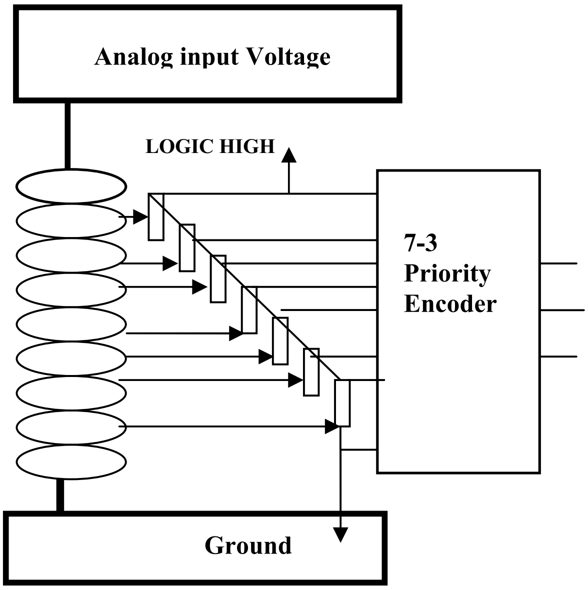
The only solution in which $K_e=K_m$ is when $x= -X_o/3$. This states that the relative displacement of the plates from its rest position is one third of the original spacing at the critical pull-in voltage irrespective of the actual mechanical force constant or actual pull-in voltage value. Substituting this displacement in the pull-in voltage equation yields

$$V_p^2 = 4X_o^2\ K_m/9C$$

or $$V_p= 2X_o/3\ \ [K_m/1.5\ C_o]^{1/2}$$

3. ANALOG-TO-DIGITAL CONVERTER

The electrostatic field force within two plated capacitor is used to move the spring contacts at 8 different locations according to the pull-in voltages found from Figure (3), each one is a multiple of the previous contacts ones. This constitutes the binary digital values. Once connected, these contacts apply a zero voltage on a PMOS switch, thus continuing the circuit to the next contact and finally to the reference high voltage. This is depicted in Figure (4), which also shows the use of 8-3 priority encoder converting the contacts tapped voltages to binary digital voltage. Table (I) lists the truth table of the encoder.



4. DIGITAL-TO-ANALOG CONVERTER

Figure (4) ADC using 7 PMOS switches at spring taps and an 7-3 priority encoder

Table (I)

7-to-3 Priority Encoder

Input							Output		
I_6	I_5	I_4	I_3	I_2	I_1	I_O	O_2	O_1	O_O
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	1
0	0	0	0	0	1	1	0	1	0
0	0	0	0	1	1	1	0	1	1
0	0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	1
0	1	1	1	1	1	1	1	1	0
1	1	1	1	1	1	1	1	1	1

In a similar manner, the two plated capacitor is used with a decoder and a spring operated potentiometer to implement a DAC. In this case a 3-8 decoder is used to energize one output at a time. This output is spring position switch which enables a current source to flow in the spring resistance thus dropping an output voltage according to I X R value, with the help of NMOS switches.

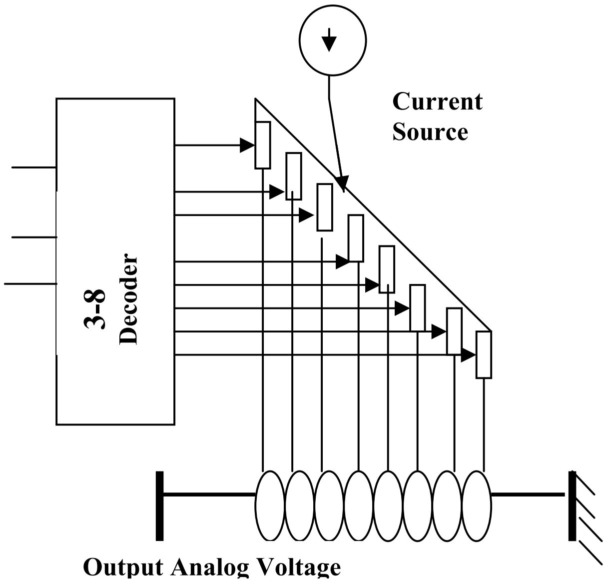


Figure (5) DAC using 8 NMOS switches at spring tap

CONCLUSION

It has been demonstrated that MEMS analog-to-digital and digital-to-analog converters can be formed using the two plated capacitor with damping spring arrangement. A 3-bit arrangement has been proposed which can be upgraded to 8-bit or more converters with cascaded decoders and encoders. 3-8 Decoders and 8-3 priority encoders are used with MOS switches and contactors in the design.

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