King Fahd University of Petroleum and Minerals
College of Computer Science and Engineering
Computer Engineering Department

COE 202: Digital Logic Design (3-0-3)
Term 181 (Fall 2018)
Major Exam 1
Saturday, October 6th, 2018
10 am

Time: 90 minutes, Total Pages: 6

SOLUTION

Notes:
Do not open the exam book until instructed
Calculators are not allowed (*basic, advanced, cell phones, etc.*)
Answer all questions
All steps must be shown
Any assumptions made must be clearly stated

Question	Maximum Points	Your Points
1	15	
2	18	
3	12	
4	15	
Total	60	

Question 1 (15 Points): Fill in the spaces for parts (1) to (7), **1 point each**.

1) Counting the number of days in a year in BCD requires (how many) _____12_____ bits.

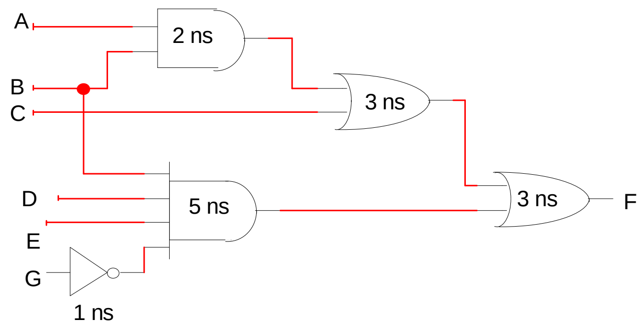
2) The largest **2-digit** number in hexadecimal is _FF______, which is equal to _255___ in decimal.

3) If the number **(31)r** in radix **r** is equal to **16** in decimal, then the radix **r =** _____5____.

4) If 10 bits are used to code only 1000 decimal values then the number of unused binary codes will be _2^{10} -1,000 = ___1024 – 1000 = 24_______.

5) If a computer uses 3 bits for red colors, 3 bits for green, and 3 bits for blue, then the total number of colors that represent all combinations of red, green, and blue is ______2^9 = 512_____.

6) For the Logic Diagram Shown:
The gate delays are shown on the gates



a) The logic function F = AB + C + BDEG' (without any re-arrangement)

b) This circuit has __3___ number of logic levels (Fill in the space)

c) The longest path's (i.e. critical path) delay = __9ns___

7) Convert between different number systems. Fill-in the table below. **(6 points)**

Decimal	Binary	Hexadecimal
123	**111 1011**	**7B**
27.625	**11011.101**	**1B.A**
0.3125	**0.0101**	**0.5**

<u>Question 2 (18 points):</u>

a) For the questions below, fill in the spaces:

1. The ASCII code 101 0001 corresponds to “Q” character. An odd parity bit is appended at the end of the code and then transmitted. The transmitted code is __101 0001 0___. The receiver receives the following code word 1110 0011, would the receiver detect an error _____No______.(Yes/NO?). **(2 points)**

2. A machine uses the following unsigned binary number representation $(XXXX.XXX)_2$, where 4 digits are used for the integer part and 3 digits are used for the fraction part. For this machine, the smallest nonzero fraction is equal to ____2^{-3}____(in decimal value), while the largest number is equal to __$2^4 - 2^{-3}$___(in decimal value). **(2 points)**

3. The minimum number of binary digits required to assign unique binary codes for the 100 students in an academic department is equal to ___7__ (how many?). When the number of students increases by a factor of 16 over the next two years, the minimum number of bits required will be ___11___(how many?). **(2 points)**

b) The decimal number 103 is to be stored in a computer’s memory. Specify how the number is represented for each of the following codes: **(3 points)**

a) BCD 5421: 0001 0000 0011

b) BCD Excess-3: 0100 0011 0110

c) Perform the following arithmetic operations in the respective numbering system. Specify the carry-in and borrow digits at every stage. **(9 points)**

110 ← Carry in $(32A)_{16}$ + $(2E8)_{16}$ --------- $(612)_{16}$	$(1011\ 0110)_2$ X $(110)_2$ ---------------- 00000000 10110110 10110110 ------------------ $(10001000100)_2$
0111 1110 ← Borrows $(1100\ 1000)_2$ - $(1001\ 1111)_2$ -------------- $(0010\ 1001)_2$	

<u>Question 3 (12 Points):</u>

1. Find the complement of F : $F = [(\bar{A} + B)CD + EG]$ **(3 points)**

$$\overline{F} = (A\overline{B} + \overline{C} + \overline{D})(\overline{E} + \overline{G})$$

2. Given F = C'D' + AB'CD'+ ABC'D' + A'C'D + AB'C Simplify F to minimum number of literals in Sum-of-Product format (SOP) **(4 points)**

F = C'D' + AB'CD'+ ABC'D' + A'C'D + AB'C = C'D' (1+AB) + AB'C(D' +1) +A'C'D

= C'D' + AB'C + A'C'D =C'(D'+A'D) +AB'C = C'(D'+A') +AB'C= C'D' + C'A' +AB' C

3. Given F (A.B,C) = ∑m(0,4,5) Simplify F to minimum number of literals in **Product-of-Sum format** (POS) **(5 points)**

First write F as an algebraic POM: F = ∏M(1,2,3,6,7)

= (A+~~B~~+C')(~~A~~+B'+C)(~~A~~+B'+C')(A'+B'+~~C~~)(A'+B'+~~C'~~) =

= (A+C') (A+B')(A'+B') = (A+C')(B'+AA')=(A+C')B' or right away recognizing that B' is common to all combinations of A&C:
= (A+C')[B'+~~(A+C)(A+C')(A'+C)(A'+C')~~] = (A+C')B'

Another solution:

F = (A+~~B~~+C')(~~A~~+B'+C)(~~A~~+B'+C')(A'+B'+~~C~~)(A'+B'+~~C'~~) =

= (A+C') (B'+C) (A'+B')
= (A + C')(B' + A')(B' + C) just reordering the terms
= (A + C')(B' + A'C) Distributive property
= (A + C')(B' + (A+C')') DeMorgan's
= (A+C') B' + (A+C')(A+C')' Distribute
= (A+C') B' 3 literals only

<u>Question 4 (15 points):</u> In this question, we denote Minterm number i by m_i and Maxterm number i by M_i.

1. Complete the following truth table where A, B, and C are the inputs. If you believe that any of the minterms or maxterms cannot be evaluated, **explain why** and leave it blank. **(2 Points)**

A	B	C	m_2	M_2	m_6	M_8
0	0	0	0	1	0	
0	0	1	0	1	0	
0	1	0	1	0	0	
0	1	1	0	1	0	
1	0	0	0	1	0	
1	0	1	0	1	0	
1	1	0	0	1	1	
1	1	1	0	1	0	

Not possible to evaluate M_8 because its subscript exceeds the maximum row index of the truth table.

2. What is the value of $M_i m_i$? Explain your answer. **(2 Points)**

$M_i m_i$ is always equal to 0 since $m_i = \overline{M_i}$.

3. Let $F(A,B,C) = \overline{A}B + A\overline{C}$. Express $F(A,B,C)$ as a sum of minterms using the $\sum$ notation. **(2 Point)**

$$F(A,B,C) = \sum m(2,3,4,6)$$

4. Let

$$G(a,b,c) = \left(\sum m(2,6)\right) + \left(\prod M(1,2,3,4,5,7)\right)$$

Express $G(a,b,c)$ **<u>algebraically</u>** as a sum of minterms. **(3 Points)**

$$G(a,b,c) = \left(\sum m(2,6)\right) + \left(\sum m(0,6)\right) = \sum m\,(0,2,6) = \overline{a}\overline{b}\overline{c} + \overline{a}b\overline{c} + ab\overline{c}$$

5. Let $F(w,x,y,z) = wz + \overline{x}\,\overline{z} + x\overline{y}$. Express $F(w,x,y,z)$ **algebraically** as a product of maxterms. **(3 Points)**

$$F(w,x,y,z) = \sum m(0,2,4,5,8,9,10,11,12,13,15)$$
$$= \prod M(1,3,6,7,14)$$
$$= (w+x+y+\overline{z})(w+x+\overline{y}+\overline{z})(w+\overline{x}+\overline{y}+z)(w+\overline{x}+\overline{y}+\overline{z})(\overline{w}+\overline{x}+\overline{y}+z)$$

6. Let $G = AB + \overline{C}D$. Express G in a product of sums standard form. **(3 Points)**

Take the dual of G

$$\begin{aligned} G_d &= (A+B)(\overline{C}+D) \\ &= A\overline{C} + AD + B\overline{C} + BD \end{aligned}$$

Take the dual of the dual

$$G = (A+\overline{C})(A+D)(B+\overline{C})(B+D)$$