King Fahd University of Petroleum and Minerals
College of Computer Science and Engineering
Computer Engineering Department

COE 202: Digital Logic Design (3-0-3)
Term 172 (Spring 2018)
Major Exam 1
Saturday Feb. 24, 2018

Time: 90 minutes, Total Pages: 9

Name:_KEY_____ ID:_____ Section: _____

Notes:

- Do not open the exam book until instructed
- **No Calculators are allowed** (*basic, advanced, cell phones, etc*.)
- Answer all questions
- All steps must be shown
- Any assumptions made must be clearly stated

Question	Maximum Points	Your Points
1	19	
2	15	
3	16	
Total	50	

Question 1. **(19 points)**

a) The minimum number of bits required to store 20 different colors in binary is equal to **5** bits. If the number of colors is multiplied by 4 (i.e. 80 colors), then the minimum number of required bits will be equal to **7** bits. (**2 points**)

b) Counting the number of seconds in one minute in **BCD** requires (how many) **8** bits. (**1 points**)

c) The smallest **non-zero** 3-bit fraction in **binary** is **$(0.001)_2$**, and its **decimal** value is equal to **$2^{-3} = 0.125$**. (**2 points**)

d) The 8-bit binary code for character "C" is **01000011**. Using **even parity**, the sender inserts an extra parity bit equal to **1**. The receiver receives a 9-bit binary code equal to **001000011**, would the receiver detect an error (Yes/No)? **YES** (**2 points**)

e) Convert between different number systems. Fill-in the table below with different representations of a number. (**6 points**)

Decimal	Binary	Hexadecimal
102.25	01100110.01	66.4
43.75	101011.1100	2B.C

f) Convert between BCD 8421 and Excess-3 codes: **(2 Points)**

BCD 8421	Excess-3 BCD
1001 0101	1100 1000
0111 0100	1010 0111

g) Given the following 8-bit binary numbers: **(4 Points)**

$A = (00110110)_2$
$B = (11101101)_2$

i) Compute **A+B** in binary and indicate whether there is a final carry.

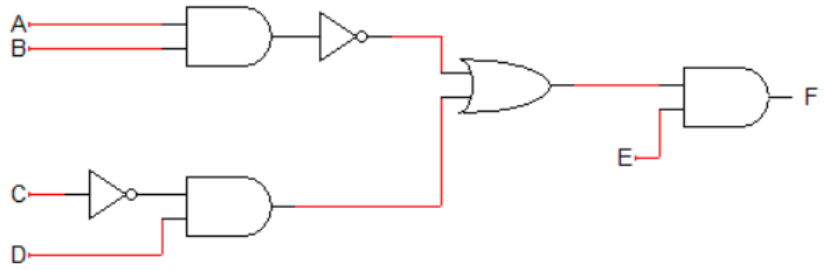
```
Carry  1 11111        Yes, there is a final carry
    A =  00110110
  + B =  11101101
       ----------
       00100011
```

ii) Compute **A-B** in binary and indicate whether there is a final borrow.

```
Borrow 1 1  1  1      Yes, there is a final borrow
    A =  00110110
  - B =  11101101
       ----------
       01001001
```

Question 2. **(15 points)**

a) Express the Boolean function, F, represented by the circuit given below in sum of products form: **(3 Points)**



F = E (C' D + (A B)') = E (C' D + A' + B') = C' D E + A' E + B' E

b) Find the complement of the function $F1 = (A\,\bar{B} + C)\bar{D} + E$ **(2 Points)**

F1 = [((A B') + C) D'] + E

F1' = [((A B') + C) D']' . E' = [((A B') + C)' + D] . E' = [(A' + B) C' + D] . E'

c) Using Boolean Algebraic manipulations, minimize the following functions to minimum number of literals in sum of products representation:

(i) $F2 = A + \bar{A}\,C + (A + C)(\bar{A} + \bar{C})$ **(3 Points)**

F2 = A + C + A C' + A' C by distributive law
= A + C by absorption

(ii) $F3 = A\,B + \bar{A}\,C + \bar{B}\,C + A\,\bar{C}$ **(4 Points)**

F3 = A B + A' C + B' C + A C' + B C — by consensus
= A B + A' C + C + A C' — by distributive law
= A B + C + A C' — by absorption
= A B + C + A — by distributive law
= A + C — by absorption

(iii) $F4 = (A + B)(A + C)(\bar{A} + \bar{B})(A + \bar{C})$ **(3 Points)**

We first take the dual of F4 and we get:
A B + A C + A' B' + A C' = A B + A' B' + A — by distributive law
= A + A' B' — by absorption
= A + B' — by distributive law

Then, we take the dual again, this leads to F4 = A B'

Question 3. **(16 points)**

a) For the following subparts (i – iv), assume that $F(A,B,C) = \sum(1,2,5,7)$

(i) Complete the truth table of the function F. **(2 Points)**

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

(ii) List all the maxterms of F using the $\prod$ notation. **(2 Points)**

$$F(A,B,C) = \prod(0,3,4,6)$$

(iii) List all the minterms of $\overline{F}$ using the Σ notation. **(2 Points)**

$$\overline{F}(A,B,C) = \Sigma(0,3,4,6)$$

(iv) Express $\overline{F}$ **algebraically** as a product-of-maxterms. **(2 Points)**

$$\overline{F}(A,B,C) = \prod(1,2,5,7) = (A+B+C')(A+B'+C)(A'+B+C')(A'+B'+C')$$

b) Let the function $G(A,B,C) = 1$ whenever $A = \overline{B}$, regardless of the value of C. List the minterms of G using the Σ notation. **(2 Points)**

$$G(A,B,C) = \Sigma(010_2, 011_2, 100_2, 101_2) = \Sigma(2,3,4,5)$$

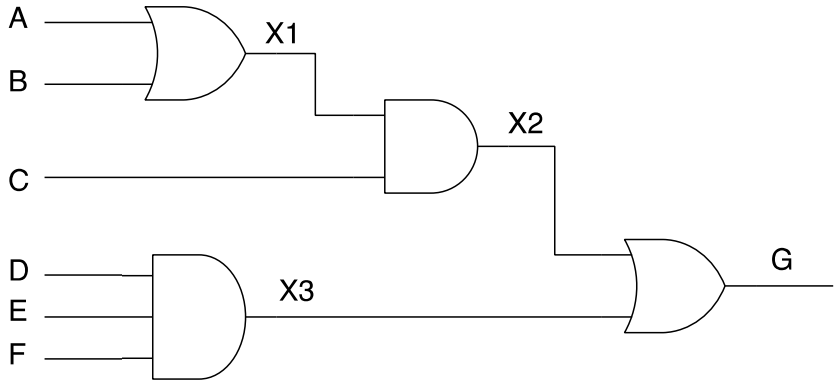
c) Let $H(A,B,C) = (\sum(1,3,5,7))(\prod(2,4,6,7))$. List the minterms of H using the Σ notation. **(2 Points)**

Since $\prod(2,4,6,7) = \sum(0,1,3,5)$, we can rewrite $H(A,B,C) = (\sum(1,3,5,7))(\sum(0,1,3,5))$. Note that H will be 1 for a given combination of A,B, and C *iff* both sums in the RHS are 1, i.e., take the intersection of the indices of both sums to get the minterms of H ➔ $H(A,B,C) = \sum(1,3,5)$.

d) Let $K(A,B,C) = AB + \bar{C}$. List the minterms of K using the Σ notation. **(2 Points)**

K(A,B,C) = AB(C+C') + C'
= ABC+ABC'+C'(A+A')
= ABC+ABC'+AC'(B+B')+A'C'(B+B')
= ABC+ABC'+ABC'+AB'C'+A'BC'+A'B'C'
= $\Sigma(0,2,4,6,7)$

e) Consider the following circuit: **(2 Points)**



The following table summarizes the propagation delay of the gates:

Gate	Delay
2-input AND gate	2 ns
3-input AND gate	3 ns
2-input OR gate	2 ns

Calculate the propagation delay for each of the following gates: X1, X2, X3, and G. The delay must be calculated from the primary inputs to the output of the gate. Please fill in the second column of the following table for answering this part.

Gate	Propagation delay (ns)
X1	2 ns
X2	4 ns
X3	3 ns
G	6 ns