King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering

Computer Engineering Department

COE 202: Digital Logic Design (3-0-3)
Term 171 (Fall 2017)
Major Exam 1
Saturday, October 21st, 2017

Time: 90 minutes, Total Pages: 6

Name:________________________________ **ID:**____________________ **Section:** _______

Notes:
Do not open the exam book until instructed
Calculators are not allowed (*basic, advanced, cell phones, etc.*)
Answer all questions
All steps must be shown
Any assumptions made must be clearly stated

Question	Maximum Points	Your Points
1	12	
2	6	
3	6	
4	6	
5	20	
Total	50	

Key

Question 1: Fill in the Spaces: **[12 points]**

1. The number of bits required to provide distinct binary codes for 60 different colors is equal to _____**6**_______ bits. If the number of colors in (i.e. 60 colors) is doubled four times (i.e. multiplied by 16), then the number of required bits will be equal to _____**10**________ bits. (2 points)

2. Counting the number of hours in one day in BCD requires a minimum of ______**8**_____ (how many) **bits**. (2 points)

3. The number $(B3D.C6)_{16}$ converted to binary is equal to _____**101-100-111-101.110-001-10**_________ while if converted to octal it will be ______**5475.614**___________. (4 points)

4. The largest decimal value for an unsigned 3-bit binary fraction number is equal to **$1-2^{-3}$ = 0.875**_________ while the smallest decimal value for an unsigned 3-bit binary fraction number is equal to **2^{-3} = 0.125**________. (2 points)

5. The ASCII code 100 0001 corresponds to "A" character. An even parity bit is appended at the end of the code and then transmitted. The transmitted code is ____**100 0001 0**_____. The receiver receives the following code word 1000 0001, would the receiver detect an error _____**No**______ (Yes/No?). (2 points)

Question 2. **(6 Points)**

The binary number 1001 0100 is stored in a computer. What is the decimal value represented if the stored number is"

a) BCD 5421 (2 points)

BCD 5421 → 64 {1 pt per digit}

b) Excess 3 BCD number (2 points)

excess-3 → 61 {1 pt per digit}

c) Unsigned binary number (2 points)

Binary unsigned → 128+16+4 = 148 {2 points}

Question 3. Perform the following arithmetic operations in the specified number system. **(6 Points)**

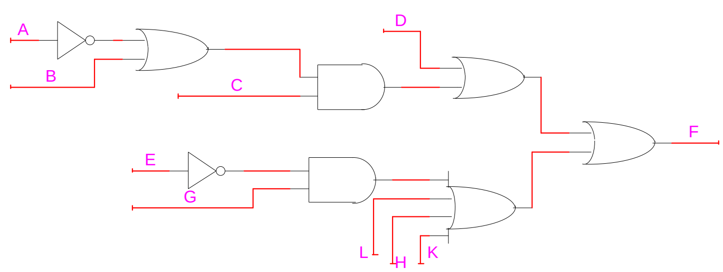
Show the details of all your work (carries, borrows …etc.)

Hexadecimal Addition	Binary Subtraction	Binary Multiplication
11 13A + E9 --------------- **223**	1 1 1 110 001 - 100 111 ----------------- **001 010**	1101 × 110 ---------------- **0000** **11101** **1101** -------------- **1001110**

{1 pt per bit/digit}

Question 4. **(5 Points)**

1. For the Logic Diagram Below:



a) **Write the Boolean expression for the output F = (A'+B)C + D + (E'G + L + H + K) (2nd set of brackets is optional)**
(as in the logic diagram without any re-arrangement) **(3 Point)**

b) This circuit has ___**5**___ number of logic levels (Fill in the space) **(1 Point)**

c) For the gates delays shown in the Table below, the worst case delay (i.e. critical path delay) of this circuit is = __**12ns**___ **(2 Point)**

Gate	Delay (in Nano seconds)
NOT	1 ns
2-IP AND	2 ns
2-IP OR	3 ns
4-IP OR	5 ns

Question 5. **(20 Points)**

1) Given the function **F(a,b,c) = a (b' + c)**,

(a) Express **F** as a product of Maxterms (use the mathematical notation $F = \prod \ldots$) **(2 points)**

(b) Express **F** as an algebraic sum of Minterms (i.e. write F as a Boolean expression) **(2 points)**

(a) F(a,b,c)= ∏M (0, 1, 2, 3, 6).

The solution using a truth table:

a	*b*	*c*	*f = a (b' + c)*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Another way ..
F is already in POS form, so obtain the Maxterms in each OR term:

a = (a+b+c), (a+b+c'), (a+b'+c), (a+b'+c') = M_0, M_1, M_2, M_3

b'+c= (a+b'+c), (a'+b'+c) = M_2, M_6

Hence, F(a,b,c)= ∏M (0, 1, 2, 3, 6).

(b) F(a,b,c) = ∑m (4,5,7) = ab'c' + ab'c + abc

2) Given the function **G(a,b,c) = ∏M (0, 1, 2)** , Express $\overline{\mathbf{F}}$ **+ G** as Product of Maxterms **(3 points)**

G = ∑m(3,4,5, 6,7) , F'(a,b,c) = ∑m(0, 1, 2, 3, 6) → F'+G is the union of their Minterms
= ∑m(0, 1, 2, 3, 4,5,6,7) = 1 (i.e. F'+G is always 1)

i.e. F'+G = ∏M (Φ)

3) **<u>Using DeMorgan's theorem</u>**, find the complement of the following two functions: **(6 points)**

a) $f = a\, b\, d' + b'\, c' + a'\, c\, d$

b) $g = (a + b)\,(b' + c) + d'\,(a' + b\, c)$

a) $f\,' = (\, a' + b' + d\,)\,(\, b + c\,)\,(\, a + c' + d'\,)$
b) $g' = (\, a'\, b' + b\, c'\,)\,(\, d + a\,(\, b' + c'\,)\,)$
{2 pts for missing first pair of () or second pair of () – operation precedence!}

4) Given that: $a\, b\, c' + a\, b'\, c + a'\, b\, c + a\, b\, c = a\, b + a\, c + b\, c$, then use the **Duality Principle to find out** $(\, a + b + c'\,)\,(\, a + b' + c\,)\,(\, a' + b + c\,)\,(a + b + c) = ???$ **(i.e. use duality to find the right hand expression) (1 points)**

$(\, a + b + c'\,)\,(\, a + b' + c\,)\,(\, a' + b + c\,)\,(a + b + c) = (a + b)\,(a + c)\,(b + c)$

5) Using the properties of Boolean algebra, minimize the following functions to the stated number of literals (Show your steps and the properties that you used): **(6 points)**

(a) F = a b' c' + a' b' c + a b' c + b c **(minimize to three literals)**

= a b' c' + a' b' c + a b' c + b c	
= a b' c' + a b' c + a b' c + a' b' c + b c	***Commutativity and Idempotence***
= a b' (c' + c) + (a + a') b' c + b c	***Distributivity***
= a b' + b' c + b c	***Complement (c' + c) = 1***
= a b' + (b' + b) c	***Distributivity***
= a b' + c	

(b) G= (x' + z) (x + y' + z) (x + y + z) **(minimize to one literals)**

= (x' + z) (x + z+y'y)	***Distributivity***
= *(x' +z) (x+ z)*	***(y' y)* = 0, 0+x+z)=(x+z)**
= z+*x' x*	***Distributivity***
= *z + 0 = z*	**Reduced to 1 literal**

Or

G= (x' + z) (x + y' + z) (x + y + z)

= (x' + z) (x + z)	**minimization theorem (x+z+y')(x+z+y) = (x+z)**
= z	**minimization theorem (z+x')(z+x) = z**

Many other solutions are also possible for (a) and (b)