King Fahd University of Petroleum and Minerals
College of Computer Science and Engineering
Computer Engineering Department

COE 202: Digital Logic Design (3-0-3)
Term 191 (Fall 2019)
Major Exam 1
Saturday Oct. 12, 2019

Time: 120 minutes, Total Pages: 9

Name:_KEY______________________________ ID:_________________ Section: _______

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Notes:

- Do not open the exam book until instructed
- **No Calculators are allowed** (*basic, advanced, cell phones, etc.*)
- Answer all questions
- All steps must be shown
- Any assumptions made must be clearly stated

Question	Maximum Points	Your Points
Q1	20	
Q2	16	
Q3	14	
Q4	15	
Total	65	

Question 1. **(20 points)**

a) **(2 points)** Counting the number of days in a month in binary requires (how many?) __5___ bits, whereas counting the same in BCD requires (how many?) __8___ bits.

b) **(2 points)** The largest octal number which has 3 integer digits and 2 fraction digits is (in octal) _777.77_, and it equals (formula is enough, no need for the final number) __$8^3 - 8^{-2}$__ in decimal.

c) **(2 points)** The decimal number 17 can be represented in BCD as _0001_0111_ and in Excess-3 as ____0100_1010______.

d) **(1 point)** In the equation $\sum_{n=0}^{5}(7 \times 8^n) = 8^m - 1$, the value of m must equal to __6____.

e) **(6 points)** Fill-in the table below with different representations of a number.

Decimal	Binary	Hexadecimal
154	**10011010**	**9A**
29.25	**00011101.0100**	**1D.4**
0.125	**0.001**	**0.2**

f) **(2 points)** If you type the word 'BC' on your keyboard, what is the binary sequence sent to the computer using 8-bit ASCII with the 8th most-significant bit being an **even parity** bit. Note that the 7-bit ASCII code of 'A' in hexadecimal is 41.

The sequence, in hexadecimal and without parity bits, is 42 , 43.
Its binary representation is 100 0010, 100 0011.
Adding even parity bits in the most-significant position, we get the new sequence:
0100 0010 , 1100 0011.

g) **(2 points)** Given that d_0, d_1, and d_2 are three integers whose values are between 0 and 15 (inclusive), find the values of d_0, d_1, and d_2 in this equation $d_0 + 16d_1 + 256d_2 = 2049$.

This is a decimal to hexadecimal conversion problem. The hex digits of 2049 are the required integers. Note that $2049 = 2^{11} + 1 = 2^3 \times 2^8 + 1 = 8 \times 16^2 + 1$, and thus $d_0 = 1$, $d_1 = 0$, and $d_2 = 8$.

h) **(1 point)** Compute $(01111111)_2 + (11011111)_2$ and indicate whether there is a carry out or not.

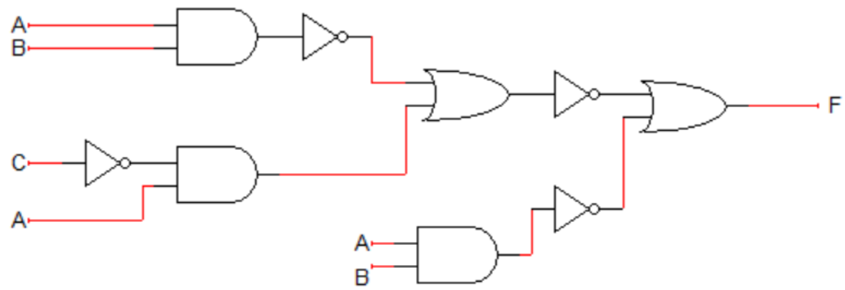
```
 11111111    ← carries
  01111111
+ 11011111
-------------
  01011110
```

i) **(2 points)** Compute $(11110000)_2 - (10101101)_2$.

```
 00001111    ← borrows
  11110000
- 10101101
--------------
  01000011
```

Question 2. **(16 points)**

a) **(3 points)** Express the Boolean function, F, represented by the circuit given below in minimal sum of products form:



F = [(A B)’ +A C’]’ + (A B)’
  = [(A B) (A’ + C)] + (A’ + B’)
  = A B C + A’ + B’= A’ + B’ + C

b) **(3 points)** Find the complement of the function $F1 = ((AB')' + C)D' + A\,C'$ and express it in minimal sum of products form.

F1 = [((A B’)’ + C) D’] + (A C’)

F1’ = [((A B’)’ + C) D’]’ . (A C’)’
  = [((A B’)’ + C)’ + D] . (A’ + C)
  = [((A B’) . C’) + D] . (A’ + C) = A’ D + C D

c) **(10 points)** Using Boolean Algebraic manipulations, minimize the following functions to minimum number of literals in sum of products representation. Show your work clearly step by step indicating the used properties of Boolean Algebra:

(i) (**2 points**) $F2 = (A + C)' + (A + C)(A' + C')$

F2 = (A + C)’ + (A’+ C’)	by simplification
= A’ C’ + A’+ C’	by Demorgan law
= A’+ C’	by absorption

(ii) **(4 points)** $F3 = A'B' + B'C + AB'C' + AB$

F3 = A' B' + B' C + A [B' C' + B]	by distributive law
= A' B' + B' C + A [C' + B]	by simplification
= A' B' + B' C + A C' + A B	by distributive law
= A' B' + B' C + A C' + A B + A C	by consensus
= A' B' + B' C + A B + A [C' + C]	by distributive law
= A' B' + B' C + A B + A [1]	by complement
= A' B' + B' C + A	by OR identity & absorption
= B' + B' C + A	by simplification
= B' + A	by absorption

(iii) **(4 points)** $F4 = (A' + B' + C')(A + C')(B + C')(B' + C)$

We first take the dual of F4 and we get:

A' B' C' + A C' + B C' +B' C

= C' [A' B' + A + B] + B'C	by distributive law
= C' [B' + A + B] + B'C	by simplification
= C' [1 + A]+ B'C	by complement
= C' + B'C	by OR identity
= C' + B'	by simplification

Then, we take the dual again, this leads to F4 = B'C'

Question 3: **(14 points)**

a) **(1 point)** The function F, where F (A,B,C,D)=∑(2,3,6), can be expressed algebraically in canonical form as:
   a. $A'BC' + A'BC + ABC'$
   b. $A'B'C + A'CD'$
   c. $(A + B + C)(A + B + C')(A + B' + C)(A' + B + C')(A' + B' + C')$
   d. Answers (a) and (c)
   e. None of the Above.

b) **(1 point)** Refer to the following statements:
Statement 1: All canonical forms for representing a function are standard forms.
Statement 2: All standard forms for representing a function are canonical forms.
Statement 3: The canonical forms and the standard forms are unique for each function
Which of these statements is/are correct?
   a. All statements.
   b. Statement 1 only.
   c. Statement 2 only.
   d. Statement 3 only.
   e. None.

c) **(5 points)** Given $G(x,y,z) = x'y + xz + yz$.
(i) **(2 points)** Derive the truth table for function G.

X	Y	Z	G
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

(ii) **(1 point)** List all the Minterms of function G(x,y,z) using the ∑ notation.

$G(x,y,z) = \sum(2, 3, 5, 7)$

(iii) **(2 points)** write function G(x,y,z) as a product of Maxterms using algebraic form.

$G = (x + y + z)(x + y + z')(x' + y + z)(x' + y' + z)$

d) **(4 points)** Given the Boolean functions F and G as:

F (x,y,z) =∑(0,2,4,5)

G (x,y,z) = (x + y + z') (x + y' + z) (x + y' + z') (x' + y + z') :

(i) **(2 points)** List the minterms of (*F.G'*) using the ∑ notation.

G*(x,y,z)* = ∏(1, 2, 3, 5) = ∑ (0, 4, 6, 7)

G'*(x,y,z)* = ∑ (1, 2, 3, 5)

F.G' = ∑ (2, 5)

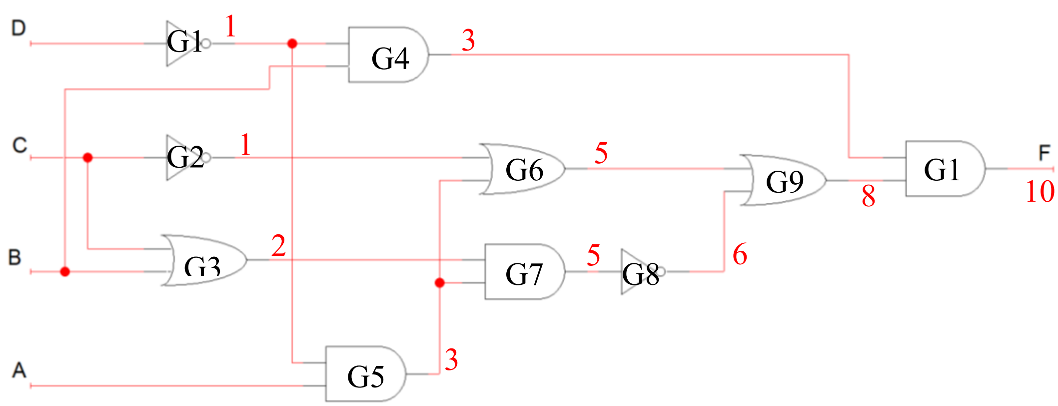
(ii) **(2 points)** List the maxterms of (*F'*+ *G*) using the ∏ notation.

F'*(x,y,z)* =∑(1,3,6,7)

G *(x,y,z)* = ∑ (0, 4, 6, 7)

F' + G =∑(0,1,3,4,6,7) = ∏(2,5)

e) **(3 points)** Given the following implementation of function F. Calculate the propagation delay of F and determine the critical path. Assume the delay of each gate is equal to the number of inputs (i.e. the delay of an inverter is 1ns, the delay of a 2-input AND/OR gate is 2ns)



The propagation delay of F = 10ns

The critical path of F is G1 – G5 – G7 – G8 – G9 – G10

Question 4. **(15 points)**

a) (**2 points**) Given the function $f(a, b, c, d) = \sum m(0, 2, 4, 6, 7, 10, 11, 12) + \sum d(1, 8, 13)$, draw the K-map of f.

Solution 4a:

f		cd			
		0 0	0 1	1 1	1 0
a b	0 0	1	X		1
	0 1	1		1	1
	1 1	1	X		
	1 0	X		1	1

b) (**13 points)** Given the following K-map of the function $g(a, b, c, d)$, where **X** is a don't-care:

g		cd			
		0 0	0 1	1 1	1 0
a b	0 0				X
	0 1	1			1
	1 1	X	1	X	1
	1 0		1	1	1

(i) (**5 points)** Write the terms of **all Prime Implicants** of g.
(ii) (**2 points)** Write the terms of **all Essential Prime Implicants** of g.
(iii) (**4 points)** Find **ALL** minimum **Sum-of-Products** expressions of g.
(iv) (**2 points)** Find **ALL** minimum **Product-of-Sums** expressions of g.

Solution i:

Five Prime Implicants: ***ab, ac, ad, bd', cd'***

Solution ii:

Two essential prime Implicants: ***ad, bd'***

Solution iii: Two Solutions

$g = ad + bd' + ac$

$g = ad + bd' + cd'$

Solution iv: Only one solution

$g = (a + d')(b + c + d)$