EE 200- Digital Logic Circuit Design
Boolean Algebra (2.1-2.4)

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Entry Questions

- What is Algebra?
- What is **Boolean** Algebra?

Objectives

1. Boolean Algebra
   - Definition
   - Theorems
   - Operator Precedence

Definition

Algebraic structure defined by a set of element, B, with two binary operators ($+$ and $\cdot$) satisfying the following:

1. The structure is closed with respect to ($+$ and $\cdot$).
2. 0 is the identity element for ($+$), and 1 is the identity element for ($\cdot$).
3. The structure is commutative with respect to ($+$ and $\cdot$).
4. ($\cdot$) is distributive over $+$ and $+$ is distributive over ($\cdot$).
5. for $x \in B$ there is $\bar{x} \in B$.
6. there exit at least two elements $x, y \in B$ such that $x \neq y$.

Properties of Boolean Algebra

1.	$x + 0 = x$	$x \cdot 1 = x$	Identity
2.	$x + x' = 1$	$x . x' = 0$	Complement
3.	$x + y = y + x$	$x \cdot y = y \cdot x$	Commutative Law
4.	$x + (y + z) = (x + y) + z$	$x \cdot (y \cdot z) = (x \cdot y) \cdot z$	Associative Law
5.	$x + (y \cdot z) = (x + y) \cdot (x + z)$	$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$	Distributive Law

Duality Principle

A principle can be obtained by interchanging AND and OR operators and replacing 0's by 1's and 1's by 0's.

- $(x + y + z + \cdots)^D = x.y.z\cdots$
- Example: $(x + 0 = x)^D = (x \cdot 1 = x)$
- $x + 1 = 1$ has the dual $x \cdot 0 = 0$
- (xy)'=x'+y' has the dual (x+y)'=x'y'

  Compare the identities on the left side with the identities on the right. Can you try to prove it by truth table?
- $(x.y + z')^D = ?$

Theorem 1

A	**B**
$$\begin{aligned} x + x &= x \\ &= (x + x) \cdot 1 \\ &= (x + x)(x + x') \\ &= x + xx' \\ &= x + 0 \\ &= x \end{aligned}$$	$$\begin{aligned} x \cdot x &= x \\ &= xx + 0 \\ &= xx + xx' \\ &= x(x + x') \\ &= x \cdot 1 \\ &= x \end{aligned}$$

Theorem 2

- **A)** $x + 1 = 1$

$$\begin{aligned} &= 1 \cdot (x + 1) \\ &= (x + x')(x + 1) \\ &= x + x' \cdot 1 \\ &= x + x' \\ &= 1 \end{aligned}$$

- **B)** $x \cdot 0 = 0$, by duality.

Theorem 3 & 4

- Theorem 3 (involution):
  $(x')'=x$.
- Theorem 4 (associative):
  $x+(y+z)=(x+y)+z$, and $x(yz)=(xy)z$.

Theorem 5

Exercise: Show the truth table for (xy)' and x'+y'

y	x	xy	(xy)'
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

y	x	y'	x'	x'+y'
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

- $\overline{+} = \cdot$
- $\overline{\cdot} = +$
- Theorem 5 (DeMorgan):
  (x+y)'=x'y', and (xy)'=x'+y'.

Theorem 6 (Absorption)

- **A)** $x + xy = x$

$$\begin{aligned} &= x \cdot 1 + (xy) \\ &= x(1 + y) \\ &= x(y + 1) \\ &= x \cdot 1 \\ &= x \end{aligned}$$

- **B)** $x(x + y) = x$, by duality.

Postulates & Theorems of Boolean Algebra

Table 2.1
Postulates and Theorems of Boolean Algebra

Postulate 2	(a)	$x + 0 = x$	(b)	$x \cdot 1 = x$
Postulate 5	(a)	$x + x' = 1$	(b)	$x \cdot x' = 0$
Theorem 1	(a)	$x + x = x$	(b)	$x \cdot x = x$
Theorem 2	(a)	$x + 1 = 1$	(b)	$x \cdot 0 = 0$
Theorem 3, involution		$(x')' = x$		
Postulate 3, commutative	(a)	$x + y = y + x$	(b)	$xy = yx$
Theorem 4, associative	(a)	$x + (y + z) = (x + y) + z$	(b)	$x(yz) = (xy)z$
Postulate 4, distributive	(a)	$x(y + z) = xy + xz$	(b)	$x + yz = (x + y)(x + z)$
Theorem 5, DeMorgan	(a)	$(x + y)' = x'y'$	(b)	$(xy)' = x' + y'$
Theorem 6, absorption	(a)	$x + xy = x$	(b)	$x(x + y) = x$

Operator Precedence

To evaluate boolean expressions:

1. Parentheses.
2. NOT.
3. AND.
4. OR.

Summary

1. Boolean Algebra
   - Definition
   - Theorems
   - Operator Precedence

Next Lecture

- Boolean Algebra