EE 200- Digital Logic Circuit Design
1.5 Complements of Numbers
1.6 Signed Binary Numbers

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Entry Question

- What is a complement of a number?
- Can you name a use for complements?

Objectives

1. Complements of Numbers
2. Addition and Subtraction of Signed Binary Numbers

Review on Complements

One's Complement:	12 = 00001100
Flip bits for -ve number	-12 = 11110011
Two's Complement:	
One's complement + 1	12 = 00001100
OR	-12 = 11110100
Toggle bits after the 1st 1 from the LSB	

Why Number's Complements?

- Complements are used to simplify the subtraction operation and for logical manipulations.
- For each base (r), there are two complements:
  1. (r-1)'s complement.
  2. r's complement, also called radix/base complement.
- For decimal numbers, there are two complements:
  1. 9's complement.
  2. 10's complement.

Complements of Decimal Numbers

Example:

For the decimal number	134795
The 9th complement is	865204
The 10th complement is	865205

Subtraction using r's complement

- To find $M - N$ in base r, we add M + r's complement of N.
- If $M > N$, the end carry must be neglected.
- If $M < N$, no end carry will result and the result is the r's complement of the answer.

Example

Subtract (76425 − 28321) using 10's complements.

The 10's complement of 28321 is 71679.

$$
\begin{array}{r}
76425 \\
+\ \underline{71679} \\
\not{1}48104
\end{array}
$$

Example

Subtract (28531 − 345920) using 10's complements.

The 10's complement of of 345920 is 654080.

$$\begin{array}{rr} & 28531 \\ + & \underline{654080} \\ \text{No end carry} \rightarrow & 682611 \end{array}$$

Therefore the difference is negative and is equal to the 10's complement of the answer, - (10's comp[682611]) = - 317389

Subtraction using (r-1)'s complement

- To find $M - N$ in base r, we add M + (r-1)'s complement of N.
- If $M > N$, the end carry must be added to the result.
- If $M < N$, no end carry will result and the result is the (r-1)'s complement of the answer.

Example

Subtract (76425 − 28321) using 9's complements.

The 9's complement of 28321 is 71678.

$$
\begin{array}{rr}
 & 76425 \\
+ & \underline{71678} \\
\text{End carry} \rightarrow & 1|48103 \\
 & \underline{\quad\quad 1} \\
 & 48104
\end{array}
$$

Signed vs. Unsigned Binary Numbers

- Unsigned binary number: All bits carries an arithmetic weight.
- Signed binary number: MSB represents the sign of the number (0=+ve, 1=-ve).

Addition of Signed Binary Numbers

Addition of signed binary numbers (M+N)

- If M and N have the same sign, add M+N and give the result the same sign, otherwise subtract and give the result the sign of the bigger number.
- In complement representation, add the two numbers including the sign bit. End carry from the sign bit is ignored. No comparison or subtraction is needed.

Examples

- Add (-6) + (+13) using signed 2's complement form with 8 bits. Repeat for (+6) + (-13).

  $(+6) \equiv 00000110$ and $(+13) \equiv 00001101$

  $(-6\ \ ) \equiv 11111010$ and $(-13\ ) \equiv 11110011$

Examples

$(+6) \equiv 00000110$ and $(+13) \equiv 00001101$
$(-6\ \ ) \equiv 11111010$ and $(-13\ ) \equiv 11110011$

$$\begin{array}{rrl} +6 \rightarrow & 00000110 & \\ -13 \rightarrow & \underline{11110011} & \\ & 11111001 & \rightarrow -7 \end{array}$$

$$\begin{array}{rrl} -6 \rightarrow & 11111010 & \\ +13 \rightarrow & \underline{00001101} & \\ & \not{1}|00000111 & \rightarrow +7 \end{array}$$

Summary

1. Complements of Numbers
2. Addition and Subtraction of Signed Binary Numbers

Next Lecture

Binary Codes