Lecture 8

Thursday, September 26, 2024 8:57 AM

Ch. 4.4

3 ways to represent a quantum ci

$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

①

$(H \otimes I)|0\rangle \otimes |0\rangle = (HI)(|00\rangle) = H|0\rangle \otimes I|0\rangle$

$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$

$= \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$

$X = \begin{matrix} & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{matrix}$

② Recall $|0\rangle \otimes |0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

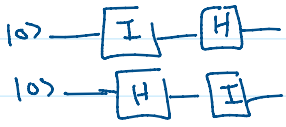
$H \otimes I = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$

$(H \otimes I)(|0\rangle|0\rangle)$

$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{matrix} 00 \\ 01 \\ 10 \\ 11 \end{matrix}$

③

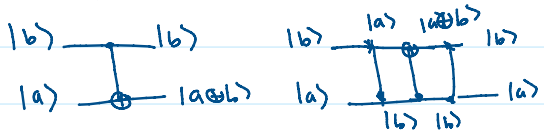


$(I \otimes H)(H \otimes I)(|00\rangle)$

Recall

$CNOT_{10}(|a\rangle \otimes |b\rangle) = |a\rangle \otimes |a \oplus b\rangle$

$CNOT_{01}(|a\rangle \otimes |b\rangle) = |a \oplus b\rangle|b\rangle$



$CNOT = (HH)\,CNOT_{01}\,(HH)$

$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$

$\propto$

$CNOT(|ab\rangle) = (HH)CNOT_{01}(HH)|ab\rangle$

$|a\rangle|a\oplus b\rangle = (HH)\overset{HH}{CNOT_{01}}(c_0|00\rangle+c_1|01\rangle+c_2|10\rangle+c_3|11\rangle)$

$= (HH)CNOT_{01}(c_0|++\rangle+c_1|+-\rangle+c_2|-+\rangle+c_3|--\rangle)$

$= (HH)CNOT_{01}\left(\frac{1}{2}(c_0+c_1+c_2+c_3)|00\rangle+(c_0-c_1+c_2-c_3)|01\rangle\right.$

$\left.+(c_0+c_1-c_2-c_3)|10\rangle+(c_0-c_1-c_2+c_3)|11\rangle\right)$

$= (HH)\frac{1}{2}\left((c_0+c_1+c_2+c_3)|00\rangle+(c_0-c_1+c_2-c_3)|11\rangle\right.$

$\left.+(c_0+c_1-c_2-c_3)|10\rangle+(c_0-c_1-c_2+c_3)|01\rangle\right)$

$c_0|++\rangle = \frac{1}{\beta}\frac{1}{2}c_0(|00\rangle+|01\rangle+|10\rangle+|11\rangle)$

$c_1|+-\rangle = \frac{1}{2}c_1(|00\rangle-|01\rangle+|10\rangle-|11\rangle)$

$c_2|-+\rangle = \frac{1}{\gamma}\frac{1}{2}c_2(|00\rangle+|01\rangle-|10\rangle-|11\rangle)$

$c_3|--\rangle = \frac{1}{\delta}\frac{1}{2}c_3(|00\rangle-|01\rangle-|10\rangle+|11\rangle)$

$= (HH)\frac{1}{2}(c_0(|00\rangle+|11\rangle+|10\rangle+|01\rangle))$

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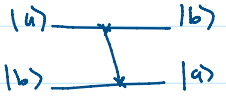
$= c_0|00\rangle+c_1|11\rangle+c_2|10\rangle+c_3|01\rangle$

$CNOT_{01}|01\rangle$

Recall swap gate



$Swap(|a\rangle|b\rangle) = |b\rangle|a\rangle$

$Swap = (CNOT)(CNOT_{01})(CNOT)|ab\rangle$

$|b\rangle|a\rangle = (CNOT)(CNOT_{01})(|a\rangle|a\oplus b\rangle)$

$(CNOT)(|a\rangle|a\oplus b\rangle)$

$|b\rangle|a\oplus b\oplus b\rangle$

$|b\rangle|a\rangle$