Lecture 6

Tuesday, September 10, 2024 8:57 AM

General one-qubit gate

- All 1-qubit gates are rotations along x,y,z axes with θ

$$\hat{n} = n_x\hat{x} + n_y\hat{y} + n_z\hat{z} \quad \text{s.t.} \quad n_x^2 + n_y^2 + n_z^2 = 1$$

$$U = e^{i\gamma}\left[\cos\left(\frac{\theta}{2}\right)I - i\sin\left(\frac{\theta}{2}\right)(n_x X + n_y Y + n_z Z)\right] \quad \text{s.t. } n_x = \frac{1}{\sqrt{2}},\ n_z = \frac{1}{\sqrt{2}}$$

$\theta = \frac{\pi}{2}$ ~~$\frac{\pi}{2}$~~ π

$$U = e^{i\gamma}\left[\cos\left(\frac{\pi}{2}\right)I - i\sin\left(\frac{\pi}{2}\right)\left(\frac{1}{\sqrt{2}}X + 0 + \frac{1}{\sqrt{2}}Z\right)\right]$$

$$= e^{i\gamma}\left[0 - i\left(\frac{1}{\sqrt{2}}X + \frac{1}{\sqrt{2}}Z\right)\right]$$

$$H = U = -ie^{i\gamma}\left[\frac{1}{\sqrt{2}}X + \frac{1}{\sqrt{2}}Z\right]$$

$$U|0\rangle = -ie^{i\gamma}\left[\frac{1}{\sqrt{2}}X + \frac{1}{\sqrt{2}}Z\right]|0\rangle = \frac{-ie^{i\gamma}}{\sqrt{2}}[X + Z]|0\rangle$$

$$= \frac{-ie^{i\gamma}}{\sqrt{2}}[X|0\rangle + Z|0\rangle] = \frac{-ie^{i\gamma}}{\sqrt{2}}[|1\rangle + |0\rangle]$$

$$= -ie^{i\gamma}\left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right] = -ie^{i\gamma}|+\rangle$$

$$U|1\rangle = -e^{i\gamma}|-\rangle$$

$|-ie^{i\gamma}|^2 = 1$

$(-ie^{i\gamma})(ie^{-i\gamma})$