Lecture 5

Sunday, September 8, 2024 8:58 AM

Quantum gates (Postulate 3 (qubit evolution))

- Quantum gates are linear
- Assume U is a quantum gate

$$U|0\rangle = \frac{\sqrt{2}-i}{2}|0\rangle - \frac{1}{2}|1\rangle \quad \} \text{ valid state}$$

$$U|1\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{2}-i}{2}|1\rangle \quad \} \sim ---$$

$$U|\varphi\rangle = U\underbrace{(\alpha|0\rangle + \beta|1\rangle)}_{\text{valid}} = U\alpha|0\rangle + U\beta|1\rangle$$

$$= \alpha\left(\frac{\sqrt{2}-i}{2}|0\rangle - \frac{1}{2}|1\rangle\right) + \beta\left(\frac{1}{2}|0\rangle + \frac{\sqrt{2}-i}{2}|1\rangle\right)$$

$$= \underbrace{\left[\alpha\left(\frac{\sqrt{2}-i}{2}\right) + \beta\left(\frac{1}{2}\right)\right]}_{\alpha'}|0\rangle + \underbrace{\left[\alpha\left(\frac{-1}{2}\right) + \beta\left(\frac{\sqrt{2}-i}{2}\right)\right]}_{\beta'}|1\rangle$$

check this is a valid state

$$|\alpha'|^2 + |\beta'|^2 = 1 = \left|\alpha\left(\frac{\sqrt{2}-i}{2}\right) + \beta\left(\frac{1}{2}\right)\right|^2 + \left|\alpha\left(\frac{-1}{2}\right) + \beta\left(\frac{\sqrt{2}-i}{2}\right)\right|^2$$

$$= \alpha^2\frac{3}{4} + \frac{\beta^2}{4} + 2\left(\cancel{\frac{\sqrt{2}-i}{4}}\right)\alpha\beta + \frac{\alpha^2}{4} + \beta^2\left(\frac{3}{4}\right) - 2\,\cancel{\frac{\sqrt{2}-i}{4}}\,\alpha\beta$$

$$= |\alpha|^2 + |\beta|^2 = 1$$

Ex 2.22

$$U|0\rangle = |0\rangle + |1\rangle \qquad |\alpha|^2 + |\beta|^2 \neq 1$$

$$U|1\rangle = |0\rangle - |1\rangle \qquad \alpha' = f(\alpha^2, \beta^2)$$

Classical Reversible Gates

Not gate is reversible

A	B
0	1
1	0

$$\text{Not}|0\rangle \to |1\rangle$$

$$\text{Not}|1\rangle \to |0\rangle$$

In general,

$$\text{Not}(\alpha|0\rangle + \beta|1\rangle) = \alpha|1\rangle + \beta|0\rangle \longrightarrow |\alpha|^2 + |\beta|^2 = 1$$

Consider this gate

A	B
0	0
1	0

$$\text{Gate}(\alpha|0\rangle + \beta|1\rangle) = \alpha|0\rangle + \beta|0\rangle = (\alpha+\beta)|0\rangle \sim |\alpha+\beta|^2 + 0 = \alpha^2 + 2\alpha\beta + \beta^2 \geq 1$$

$$i \; |0$$

$$\text{Gate}(\alpha|0\rangle+\beta|1\rangle) = \alpha|0\rangle+\beta|0\rangle = (\alpha+\beta)|0\rangle \quad \sim \quad |\alpha+\beta|^2 + 0 = \alpha^2+2\alpha\beta+\beta^2 \geq 1$$

Common Quantum Gates (1 qubit)

- Identity

$$I|0\rangle = |0\rangle \quad , I|1\rangle = |1\rangle$$

- Pauli X gate (NOT) Rotation around x

$$X|0\rangle = |1\rangle \quad , X|1\rangle = |0\rangle$$

$$X^2 = I \quad \sim \quad X^{1000} = (X^2)^{500} = I^{500} = I \quad X^{1001} = X$$

- Pauli Y gate Rotation around Y

$$Y|0\rangle = i|1\rangle$$

$$Y|1\rangle = -i|0\rangle$$

$$Y^2 = I$$

$$Z =$$

- Pauli Z gate Rotation around Z

$$Z|0\rangle = |0\rangle$$

$$Z|1\rangle = -|1\rangle$$

$$Z^2 = I$$

- Hadamard Gate

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$$

- Phase gate S ($S^2 = Z$) $\pi/4$

$$S|0\rangle = |0\rangle \quad , S|1\rangle = i|1\rangle$$

- T gate ($T^2 = S$) $\pi/8$

$$T|0\rangle = |0\rangle \quad , T|1\rangle \; e^{i\pi/4}|1\rangle$$

General 1-qubit gate

$n_x, n_y, n_z \in [0, 1]$

$$U = e^{i\gamma}\left[\cos\left(\frac{\theta}{2}\right) I - i\sin\left(\frac{\theta}{2}\right)(n_x X + n_y Y + n_z Z)\right)$$