Lecture 21

Sunday, November 3, 2024 9:02 AM

Input $f: \Sigma^n \to \Sigma \quad \Sigma \in \{0,1\}$

$U: |a\rangle|x\rangle \to |a \oplus f(x)\rangle|x\rangle$

$Z_f: |x\rangle \to (-1)^{f(x)}|x\rangle$

$OR = \begin{cases} 0 & x = 0^n \\ 1 & x \neq 0^n \end{cases}$

$Z_{OR} = \begin{cases} |x\rangle & x = 0^n \\ -|x\rangle & x \neq 0^n \end{cases}$

$2|0^n\rangle\langle 0^n| - I$

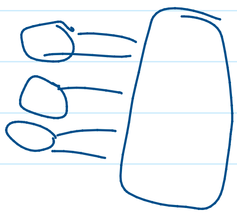
$\begin{pmatrix} 2 & 0 & 0 \\ 0 & \ddots & \\ & & \ddots \end{pmatrix} - \begin{pmatrix} 1 & & \\ & 1 & \\ & & \ddots \end{pmatrix}$

$\begin{pmatrix} 2 & \\ & \end{pmatrix} - I^{\otimes n}$

$H^{\otimes n}|0^n\rangle$

G

Measure



$G = H^{\otimes n} Z_{OR} H^{\otimes n} Z_f$

$x \in \Sigma^n, \; f(x) = 1$

$A_0 = \{x \in \Sigma^n : f(x) = 0\}$

$A_1 = \{x \in \Sigma^n : f(x) = 1\}$

$$|A_0\rangle = \frac{1}{\sqrt{|A_0|}} \sum_{x \in A_0} |x\rangle$$

$$|A_1\rangle = \frac{1}{\sqrt{|A_1|}} \sum_{x \in A_1} |x\rangle$$

$$|u\rangle = H^{\otimes n}|n\rangle$$

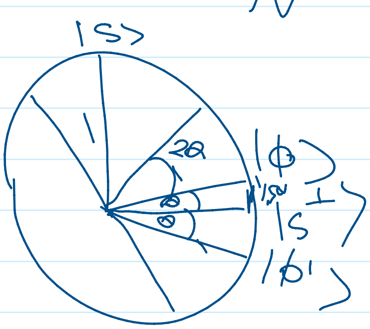
$$= \sqrt{\frac{|A_0|}{N}}|A_0\rangle + \sqrt{\frac{|A_1|}{N}}|A_1\rangle$$

$$G = H^{\otimes n} Z_{or} H^{\otimes n} Z_f$$

$$G = H^{\otimes n} Z_{0p} H^{\otimes n} Z_f$$
$$= \left(\underbrace{H^{\otimes n}\, 2|0\rangle^{\otimes n}}\, \underbrace{\langle 0|^{\otimes n} - I\, H^{\otimes n}} \right) Z_f$$
$$= \left(2|\psi\rangle\langle\psi| - I \right) Z_f$$

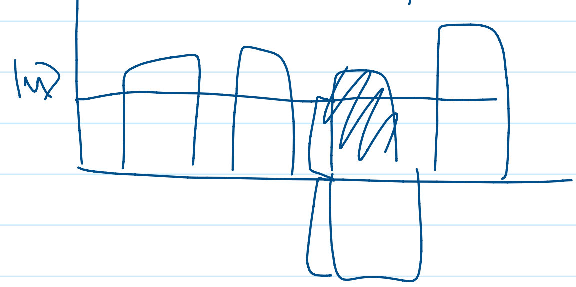
$$Z_f |A_0\rangle = |A_0\rangle$$
$$Z_f |A_1\rangle = -|A_1\rangle$$

$$G|A_0\rangle = (2|\psi\rangle\langle\psi| - I) Z_f |A_0\rangle$$
$$= (2|\psi\rangle\langle\psi| - I)|A_0\rangle$$
$$= 2|\psi\rangle\langle\psi|A_0\rangle - |A_0\rangle$$
$$= 2\sqrt{\frac{|A_0|}{N}}\, |\psi\rangle - |A_0\rangle$$
$$= 2\sqrt{\frac{|A_0|}{N}} \left(\sqrt{\frac{|A_0|}{N}}\, |A_0\rangle + \sqrt{\frac{|A_1|}{N}}\, |A_1\rangle \right) - |A_0\rangle$$
$$G|A_0\rangle = \frac{|A_0| - |A_1|}{N} |A_0\rangle + 2\sqrt{\frac{|A_0||A_1|}{N}}\, |A_1\rangle$$



$$|s^{\perp}\rangle = |s\rangle - \frac{1}{\sqrt{N}} |\phi\rangle$$

$$G|A_1\rangle = -2\sqrt{\frac{|A_1| \cdot |A_2|}{N}}\, |A_0\rangle + \frac{|A_0| - |A_1|}{N}$$

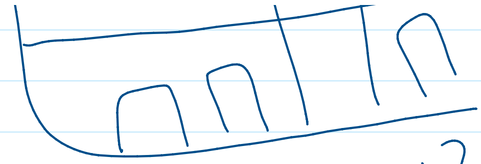


$$(1+t)\theta = \frac{\pi}{2}$$
$$(1+t)\frac{1}{\sqrt{N}} = \frac{\pi}{2}$$
$$t = \frac{\pi}{2}\sqrt{N} - 1$$
$$O(\sqrt{N})$$
$$N^{1/2}$$

$$\theta = \sin^{-1}\left(\frac{1}{\sqrt{N}}\right)$$
$$\approx \frac{1}{\sqrt{N}}$$

$O(\sqrt{N})$

$N^{1/2}$ $\quad$ N^{1}



$$M = \begin{pmatrix} \frac{|A_0| - |A_1|}{N} & -2\sqrt{\frac{|A_0||A_1|}{N}} \\ 2\sqrt{\frac{|A_0||A_1|}{N}} & \frac{|A_0| - |A_1|}{N} \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{|A_0|}{N}} & -\sqrt{\frac{|A_1|}{N}} \\ \sqrt{\frac{|A_1|}{N}} & \sqrt{\frac{|A_0|}{N}} \end{pmatrix}^2$$

$$\theta = \sin^{-1}\left(\sqrt{\frac{|A_1|}{N}}\right)$$

$$\sqrt{M} = \begin{pmatrix} \cos(\cancel{2}\theta) & -\sin(\cancel{2}\theta) \\ \sin(\cancel{2}\theta) & \cos(\cancel{2}\theta) \end{pmatrix}$$