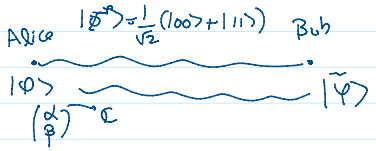
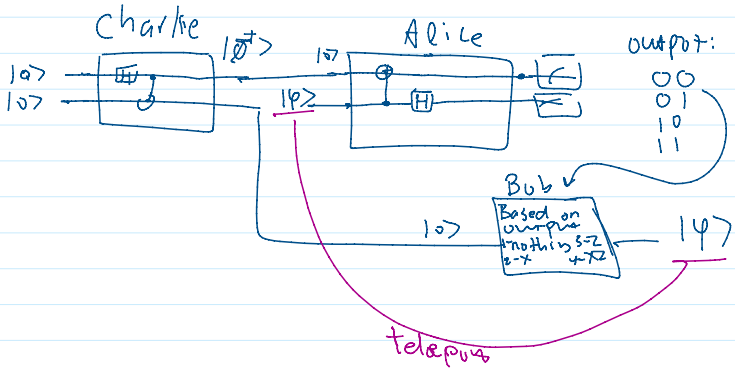
Lecture 12

Tuesday, October 8, 2024 9:34 AM

Quantum Teleportation

- This is a protocol to teleport a "state" of a qubit





Alice has 3 qubit

$$|\varphi\rangle|\beta^+\rangle = (\alpha|0\rangle+\beta|1\rangle)\left(\frac{1}{\sqrt{2}}|00\rangle+|11\rangle\right)$$
$$= \frac{1}{\sqrt{2}}[\alpha|0\rangle(|00\rangle+|11\rangle)+\beta|1\rangle(|00\rangle+|11\rangle)]$$
$$= \frac{1}{\sqrt{2}}[\alpha(|000\rangle+|011\rangle)+\beta(|100\rangle+|111\rangle)]$$

1- Alice applies CNOT on her two left most qubits

$$= \frac{1}{\sqrt{2}}[\alpha(|000\rangle+|011\rangle)+\beta(|110\rangle+|101\rangle)]$$
$$\rho$$

2- Alice applies H on her left qubit

$$= \frac{1}{\sqrt{2}}[\alpha(|+00\rangle+|+11\rangle)+\beta(|-10\rangle+|-01\rangle)]$$
$$= \frac{1}{2}[\alpha(|0\rangle+|1\rangle)(|00\rangle+|11\rangle)+\beta(|0\rangle-|1\rangle)(|10\rangle+|01\rangle)]$$
$$= \frac{1}{2}[\alpha|0\rangle|00\rangle+\alpha|0\rangle|11\rangle+\alpha|1\rangle|00\rangle+\alpha|1\rangle|11\rangle+\beta|0\rangle|10\rangle+\beta|0\rangle|01\rangle-\beta|1\rangle|10\rangle-\beta|1\rangle|01\rangle]$$
$$= \frac{1}{2}[|00\rangle(\alpha|0\rangle+\beta|1\rangle)+|01\rangle(\alpha|1\rangle+\beta|0\rangle)+|10\rangle(\alpha|0\rangle-\beta|1\rangle)+|11\rangle(\alpha|1\rangle-\beta|1\rangle)]$$

3- Alice will measure her two qubits (left), she will get one of the following states with equal prob

$|00\rangle(\alpha|0\rangle+\beta|1\rangle)$
$|01\rangle(\alpha|1\rangle+\beta|0\rangle)$
$|10\rangle(\alpha|0\rangle-\beta|1\rangle)$
$|11\rangle(\alpha|1\rangle-\beta|0\rangle)$

4- Alice sends the two bits to Bob, he will apply the following to his qubit

a) If 00, applies nothing
b) If 01, applies X
c) If 10, applies Z
d) If 11, applies XZ