Lecture 11

Sunday, October 6, 2024 8:59 AM

Bell's Inequality

Difficult to prove

CHSH Inequality

Stastical physics Vs Quantum mechanics

$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Alice ~~~~~ Bob

Four possibilities:

00, 01, 10, 11

To measure:

1. Alice measures her qubit in one of two basis
   a) $A = \{|0\rangle, |1\rangle\}$ — Z
   b) $\grave{A} = \{|+\rangle, |-\rangle\}$ — H X
2. Bob measures his qubit in one of two basis
   a) $B = \left\{\frac{1}{\sqrt{4+2\sqrt{2}}}\left[(1+\sqrt{2})|0\rangle + |1\rangle\right], \frac{1}{\sqrt{4-2\sqrt{2}}}\left[(1-\sqrt{2})|0\rangle + |1\rangle\right]\right\}$ — S H T H T
   b) $\grave{B} = \left\{\frac{1}{\sqrt{4+2\sqrt{2}}}\left[(-1-\sqrt{2})|0\rangle + |1\rangle\right], \frac{1}{\sqrt{4-2\sqrt{2}}}\left[(-1+\sqrt{2})|0\rangle + |1\rangle\right]\right\}$ — S H T† H X

Alice and Bob have four possibilties

$A B, \grave{A} B, A \grave{B}, \grave{A}\grave{B}$

Consider each case

1. AB: four outcom, with prob.

$\{AB\} = P_{00}, P_{01}, P_{10}, P_{11}$

$E[\{AB\}] = 00 P_{00} + 01 P_{01} + 10 P_{10} + 11 P_{11}$

- If $A^{\grave{B}}$ measures $|0\rangle \to +1$, otherwise $|1\rangle \to -1$

$E[\{AB\}] = P_{00} - P_{01} - P_{10} + P_{11}$

2. $A\grave{B}$

$E[\{A\grave{B}\}] = P_{00} - P_{01} - P_{10} + P_{11}$

3. Same

4. Same

X is RV

$E = \sum x P_x$

- Les S be the global Experiment

$$E[S] = E[\{A,B\}] + E[\{A,\vec{B}\}] + E[\{\vec{A},B\}] \ominus E[\{\vec{A},\vec{B}\}]$$

q_1 ?

This should be correct to accept LHV $|S| \leq 2$ $\xrightarrow{q_2}$?

- Let's try Quantum mechanics (using simulator)

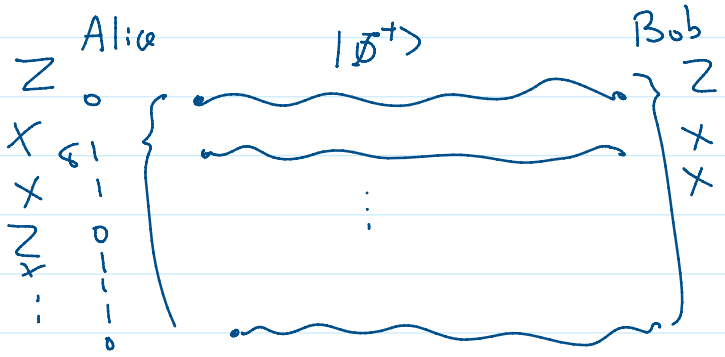
$$E[\{A,B\}] = \frac{1}{\sqrt{2}}$$
$$E[\{A,\vec{B}\}] = \frac{1}{\sqrt{2}}$$
$$E[\{\vec{A},B\}] = \frac{1}{\sqrt{2}}$$
$$E[\{\vec{A},\vec{B}\}] = \frac{1}{\sqrt{2}}$$

$$|S| = \frac{4}{\sqrt{2}} > 2$$

No-Signaling Theorem

'Q' = $(01101110)_2$

Alice wants to send 'Q' to bob



Alice chooses the measure basis based on the bit value

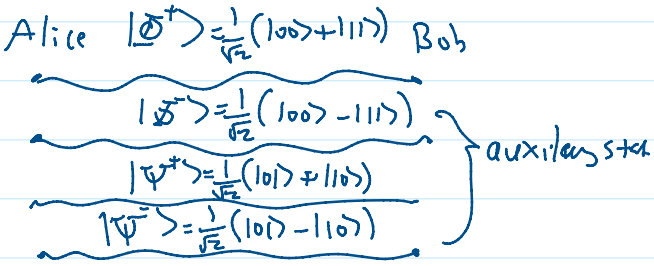
a) select Z-basis to send $|0\rangle$

b) " X-basis " " $|1\rangle$

Superdense Coding

Classically: If Alice wants to send 2 logical bits.
she needs two physical bits

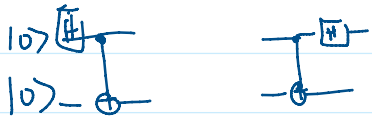
Quantum: Alice need 1 Qubit only!



To send

- 00, Alice does nothing $(|\Phi^+\rangle)$
- 01, Alice applies X gate to her qubit $(|\Psi^+\rangle)$
- 10, " ~ Z gate $(|\Phi^-\rangle)$
- 11, " ~ X and Z gates $(|\Psi^-\rangle)$

- Bob need to apply CNOT then H



$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle) = |0\rangle \otimes |+\rangle \xrightarrow{H} |00\rangle$

$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|00\rangle - |01\rangle) = |0\rangle \otimes |-\rangle \xrightarrow{H} |01\rangle$

$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|10\rangle + |11\rangle) = |1\rangle \otimes |+\rangle = |10\rangle$

$|\Psi^-\rangle = \frac{1}{\sqrt{2}}$ - — — — — — — — $|11\rangle$