# Lecture 8

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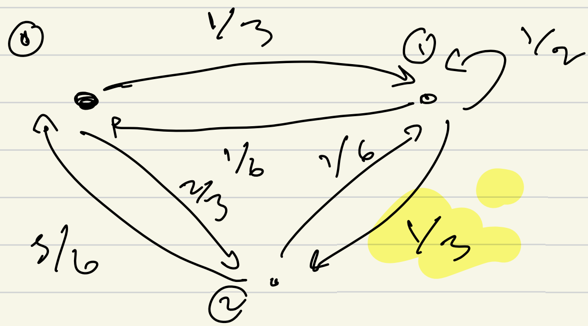
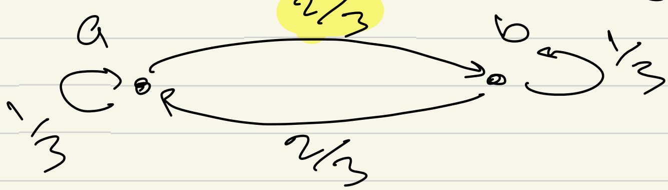
# Assembling System

Blue




$$G_m = \begin{bmatrix} 0 & 1/6 & 5/6 \\ 1/3 & 1/2 & 1/6 \\ 2/3 & 1/3 & 0 \end{bmatrix}$$

Red




$$G_n = \begin{bmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \end{bmatrix}$$

* What should wer use?

$$X = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \otimes \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0a \\ 0b \\ 1a \\ 1b \\ 2a \\ 2b \end{bmatrix} = \begin{bmatrix} 1/18 \\ 0 \\ 2/18 \\ 1/3 \\ 0 \\ 1/2 \end{bmatrix}$$

Blue - state 2 &
Red - b

$\leq 1/2$

4 $\quad 1a \xrightarrow{1/3 \times 2/3 = 2/9} 2b$

$$ij \xrightarrow{M[i i'] \, N[j j']} i' j'$$

$$G_m \otimes G_n = \begin{bmatrix} 0 & 1/6 & 5/6 \\ 1/3 & 1/2 & 1/6 \\ 2/3 & 1/3 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \end{bmatrix}$$

$6 \times 6$

$$= \begin{bmatrix} 0 & 0 & 1/18 & 2/18 & 5/18 & 10/18 \\ 0 & 0 & 2/18 & 1/18 & 10/18 & 5/18 \\ 1/9 & 2/9 & 1/6 & 2/6 & 1/18 & 2/18 \\ 2/9 & 1/9 & 2/6 & 1/6 & 2/18 & 1/18 \\ 2/9 & 4/9 & 1/9 & 2/9 & 0 & 0 \\ 4/9 & 2/9 & 2/9 & 1/9 & 0 & 0 \end{bmatrix}$$

Page 99 for a graph

* Tensor product is used to combine the states of two seperate (quantum) system
* TP is used model combined changes of the two systems by tensor product of adj. matrixes
* In quantum system, there are way ~~more~~ states than the ones in tensor product. Those are called entangled states.

* To model m "blue" marbles

$$\underline{\underline{G^{m}}} = \underbrace{G_m \times G_m \times \cdots \times G_m}_{\cancel{m\ time}\ m\ \text{times}}$$

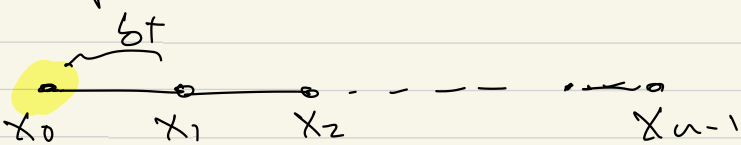
$$\boxed{M_G^{\otimes m}} = M_G \otimes M_G \otimes \cdots \otimes M_G$$

$2^m$-by-$2^m$ matrix $\qquad 2^8$-by-$2^8$

# Chapter 4

Objective: To model the quantum physical system of particles

* Consider a line with a finite number of points




$x_1 = x_0 + \delta x, \; x_2 = x_1 + \delta x, \; \ldots, \; x_{n-1} = x_{n-2} + \delta x$

* The state can be associated with a n-dimensional complex vector space

$$[c_0, c_1, \ldots, c_{n-1}]^T$$

* A particle ~~is~~ at point $x_i$ is denoted by $|x_i\rangle$ : Ket notation (col. vector)

$$|x_0\rangle \longrightarrow [1, 0, 0, \ldots, 0]^T$$
$$|x_1\rangle \longrightarrow [0, 1, 0 \ldots\ 0]^T$$
$$\vdots$$
$$|x_{n-1}\rangle \longrightarrow [0, 0, \ldots, 1]^T$$

Canonical basis of $\mathbb{C}^n$

* This is enough for classical system
* We can represent the superposition state using a linear combination of $|x_0\rangle, |x_1\rangle \ldots |x_{n-1}\rangle$

$$\underbrace{|\psi\rangle}_{\text{arbitrary state}} = c_0|x_0\rangle + c_1|x_1\rangle + \ldots + c_{n-1}|x_{n-1}\rangle$$

$c_0, c_1, \ldots, c_{n-1}$ as Complex weights

called <u>Complex amplitudes</u>

$$|\psi\rangle = [c_0, c_1, \ldots, c_{n-1}]^T$$

→ is a superposition of all state

$|x_0\rangle, |x_1\rangle \ldots$

- There are different blending of such superposition states depend. on the values of $c_0, c_1, \ldots, c_{n-1}$

- The prob. of a particle is at position $x_i$ (after measuring)

$$p(x_i) = \frac{|c_i|^2}{||\psi\rangle|^2} = \frac{|c_i|^2}{\sum_j |c_j|^2}$$

$$|\psi\rangle \xrightarrow[\text{(meas-)}]{\text{observation}} |x_i\rangle$$

$p(x_i)$