Lecture 5

14/9/202

Prop 2.6.5 (Page 65)

If U is unitary, then

$$\langle UV, UV' \rangle = \langle V, V' \rangle$$

for any $V, V' \in \mathbb{C}^n$

* $|UV| = |V|$

* Note
  - U matrix preserves the geometry
  - if U is unitary, there is a matrix $U^\dagger$ that can undo the action of U

Tensor Product

* Given two vector spaces $\mathbb{V}$ & $\mathbb{V}'$, their tensor product space $\mathbb{V} \otimes \mathbb{V}'$ is the set of "tensors" of all vector

$$\{ V \otimes V' \mid V \in \mathbb{V} \text{ and } V' \in \mathbb{V}' \}$$

* A typical component is

$$\sum_{i=0}^{P-1} c_i (V_i \otimes V_i')$$

* The dimension of $\mathbb{V} \otimes \mathbb{V}'$ is the dimension of $\mathbb{V}$ times the dimension of $\mathbb{V}'$
* $\mathbb{V} \times \mathbb{V}'$ is the vector space whose states are the states of a system $\mathbb{V}$ or $\mathbb{V}'$, or both
* $\mathbb{V} \otimes \mathbb{V}'$ is the vector space whose states are pairs of states; one from $\mathbb{V}$ and the other is from $\mathbb{V}'$

* Ex $\mathbb{C}^m$ and $\mathbb{C}^n$, Dimension $\mathbb{C}^m \otimes \mathbb{C}^n$

  mn

Def

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \otimes \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} a_0 \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} \\ a_1 \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} \\ \vdots \\ a_2 \\ a_3 \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} \end{bmatrix}$$

$4 \qquad 3$

$$= \begin{bmatrix} a_0 b_0 \\ a_0 b_1 \\ a_0 b_2 \\ a_1 b_0 \\ a_1 b_1 \\ a_1 b_2 \\ \vdots \\ a_3 b_2 \end{bmatrix}$$

* A vector that can be written as a tensor of two vectors is called separable
* — — — — not entangled

* Tensor product of matrices

$$\overset{2\times 2}{A} = \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix}, \quad \overset{3\times 3}{B} = \begin{bmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{bmatrix}$$

$$\overset{6\times 6}{\underline{A \otimes B}} = \begin{bmatrix} a_{00}[B] & a_{01}[B] \\ a_{10}[B] & a_{11}[B] \end{bmatrix}$$

6×6

$$\underline{\overset{m\times n}{\mathbb{C}} \otimes \overset{m'\times n'}{\mathbb{C}} \rightarrow \overset{mm'\times nn'}{\mathbb{C}}}$$

$$\begin{bmatrix} a_{00}b_{00} & a_{00}b_{01} & a_{00}b_{02} & \cdots \\ a_{00}b_{10} & a_{00}b_{11} & a_{00}b_{12} & \\ a_{00}b_{20} & a_{00}b_{21} & a_{00}b_{22} & \\ & \vdots & & \ddots \end{bmatrix}$$

$$\ast\ \underset{m\times m}{(A} \otimes \underset{n\times n}{B)}_{[j,k]} = A_{[j/n,\ k/m]} \times B_{[j \bmod n,\ k \bmod m]}$$

Ex

Calculate

$$\underset{3\times 3}{\begin{bmatrix} 3+2i & 5-i & 2i \\ 0 & 12 & 6-3i \\ 2 & 4+4i & 9+3i \end{bmatrix}} \otimes \underset{3\times 3}{\begin{bmatrix} 1 & 3+4i & 5-7i \\ 10+2i & 6 & 2+5i \\ 0 & 1 & 2+9i \end{bmatrix}}$$

9×9

$$\begin{bmatrix} 3+2i & 1+18i & 14-11i & \cdots \\ 26+26i & 18+12i & -4+19i & \cdots \\ 0 & 3+2i & -12+31i & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \cdots \\ & \vdots & & \end{bmatrix}$$

$$*(A \otimes B) * (V \otimes V') = A * V \otimes B * V'$$

Chapter 3 Leap from Clasical to Quantum

Objectiv

1) Cast quantum mecanics in matrics graphs
2) Introduce core ideas QP

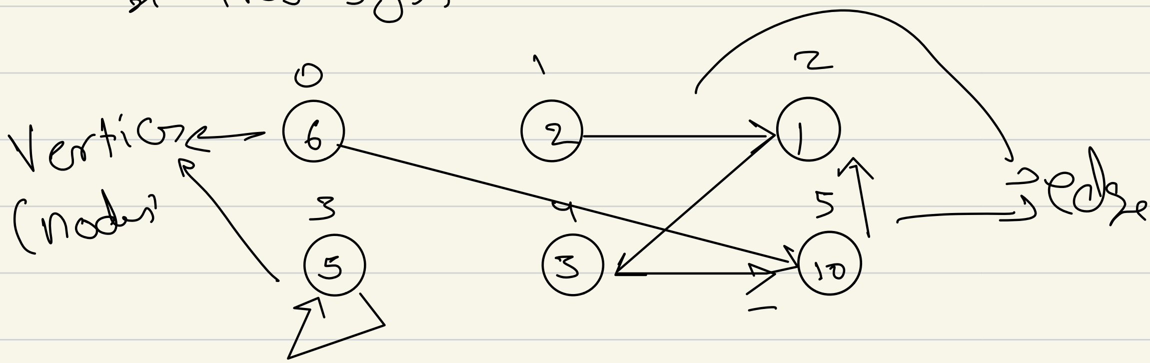
Ex

$$\begin{array}{ccc} 0 & 1 & 2 \\ 6 & 2 & 1 \\ 3 & 4 & 5 \\ 5 & 3 & 10 \end{array}$$

a) $X_1 = [6, 2, 1, 5, 3, 10]^T$ a state

b) $X_2 = [5, 5, 0, 2, 0, 15]^T$

* Find a way to describe "dynamics" of this system

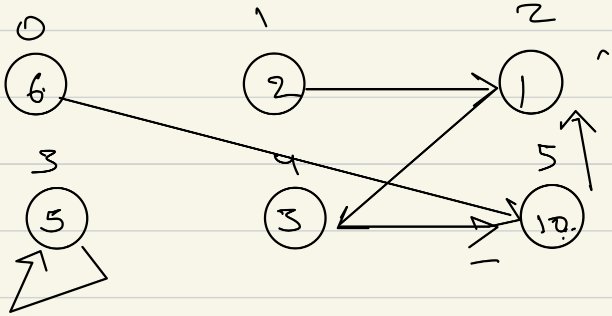


<u>Def</u> A graph $G = \{V, E\}$, where

V is a set of vertices (nodes),
E is a set of edges. $(x,y) \in E$
where $x, y \in V$

$$E \subseteq \{(x,y) \mid (x,y) \in V^2 \cancel{\& x \neq y}\}$$

* Each edge from node i to j represents a "time click" which marbles move from node i to j



* Adjancy Matrix

$$A = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} \; & \; & \; & \; & \; & \; \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \end{array}$$

$\underline{M_{[i,j]}} = 1$ if there is an edge from $\underline{j \text{ to } i}$

$$
\underbrace{\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}}_{M}
\underbrace{\begin{bmatrix} 6 \\ 2 \\ 1 \\ 5 \\ 3 \\ 10 \end{bmatrix}}_{\substack{X \\ \text{befor}}}
=
\underbrace{\begin{bmatrix} 0 \\ 0 \\ 12 \\ 5 \\ 1 \\ 9 \end{bmatrix}}_{\substack{Y \\ \text{after}}}
$$

* M is a way to describe how the marbles move from t to $t+1$

* $M^2_{[i,j]} = 1$ iff there exists a path from j to i of length 1

$$j \longrightarrow k \longrightarrow i$$

$M^k_{[i,j]} = 1$ ... a path of length k

* $Y_k = M^k X$ repr. the new syst. state after k time clicks

Ex

$$M^2 x = M(Mx) = MY_1$$

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} 0 \\ 0 \\ 12 \\ 5 \\ 1 \\ 9 \end{bmatrix}}_{Y_1} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 9 \\ 5 \\ 12 \\ 1 \end{bmatrix}}_{Y_2}$$

$$M^3 x = M(Y_2) = M(M(MY_1))$$

$$= M^3 x$$

$$\vdots$$