Lecture 13

14/10/20

* Hadamard gate

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \longrightarrow \text{Hadamard matrix}$$

- Hadamard gate creates a superposition state

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{|0\rangle} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$

* Pauli matrices

$$\sigma_a^{2\times 2} = \begin{pmatrix} \delta_{a3} & \delta_{a1} - i\,\delta_{a2} \\ \delta_{a1} + i\,\delta_{a2} & -\delta_{a3} \end{pmatrix}$$

$$\delta_{ab} = \begin{cases} +1 & \text{if } a = b \\ 0 & a \neq b \end{cases}$$

$$a, b = \underset{x}{1},\ \underset{y}{2},\ \underset{Z}{3}$$

$X = X^\dagger \quad X^\dagger X = I = X^2$

$$\sigma_1 = \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = X \longrightarrow \text{NOT}$$

± 1

$$\sigma_2 = \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = Y \qquad \pm 1$$

$$\sigma_3 = \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \pm 1$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

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4 $\sqrt{NOT}$ gate

$$\sqrt{NOT} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & + \\ 1 & 1 \end{bmatrix}$$

$$\sqrt{NOT} \neq \sqrt{NOT}^{\dagger}$$

- Why is it called $\sqrt{NOT}$?

$$\sqrt{NOT} \cdot \sqrt{NOT} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}^{|0\rangle} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 \\ 2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{|1\rangle}$$

NOT

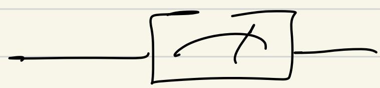
$|1\rangle$

$$\sqrt{NOT}\cdot|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}-1\\1\end{bmatrix}$$

$$\sqrt{NOT}\,\frac{1}{\sqrt{2}}\begin{bmatrix}-1\\1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}-2\\0\end{bmatrix} = \begin{bmatrix}-1\\0\end{bmatrix}$$

$$= -1|0\rangle$$

* Measurement gate



* Qubit Representation

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle$$

$$c_0 = r_0 e^{i\phi_0}$$

$$c_1 = r_1 e^{i\phi_1}$$

$$|\psi\rangle = r_0 e^{i\phi_0}|0\rangle + r_1 e^{i\phi_1}|1\rangle$$

$$e^{-i\phi_0}|\psi\rangle = r_0|0\rangle + r_1 e^{i\overset{\phi}{\cancel{(\phi_1 - \phi_0)}}}|1\rangle$$

$$|c_0|^2 + |c_1|^2 = 1 = |r_0 e^{i\phi_0}|^2 + |r_1 e^{i\phi_1}|^2$$

$$r_0^2 + r_1^2 = 1$$

$$r_0 = \cos(\theta) \quad , \quad r_1 = \sin\theta$$

$$|\psi\rangle = \cos\theta|0\rangle + e^{i\phi}\sin\theta$$

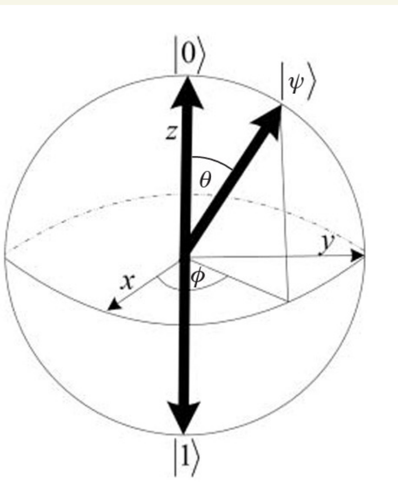


Figure 5.6. Bloch sphere.

Two parameter

θ latik

ϕ longitude

$x = \cos\phi \sin\theta$

$y = \sin\phi \sin\theta$

$z = \cos\theta$

$0 \leq \phi \leq 2\pi \quad 0 \leq \theta \leq \pi$

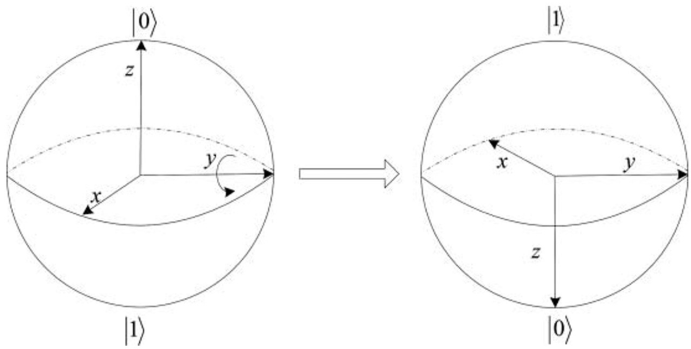
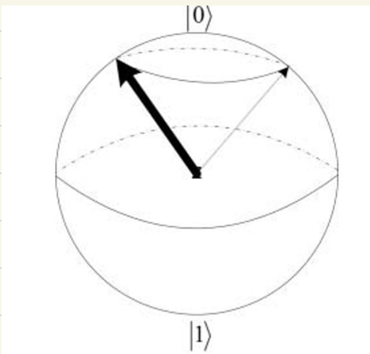


Figure 5.7. A rotation of the Bloch sphere at y.

* Phase shift gate

$$R(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{\theta} \end{bmatrix}$$



$\cos(\theta)|0\rangle + e^{i\phi}\sin(\theta)|1\rangle$

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{\theta} \end{bmatrix}\begin{bmatrix} \cos(\theta) \\ e^{i\cancel{\phi}}\sin(\theta) \end{bmatrix} = \begin{bmatrix} \cos\theta \\ e^{i\phi+\theta}\sin(\theta) \end{bmatrix}$$

Change longitud only

$$R_x(\theta) = \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2} X$$

$$R_y(\theta) = \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2} Y$$

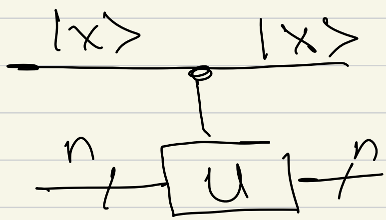
$$R_z(\theta) = \cos\frac{\theta}{2} \underline{I} - i\sin\frac{\theta}{2} Z$$

$$R_D(\theta) = \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2}(D_x X + D_y Y + D_z Z)$$

$D_x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $D_y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$D_z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

- Controlled - U (^cU)
  - Equivalent to IF-THEN



- Performs U operation if $|x\rangle = \underline{1}$
- if $|x\rangle$ is 0, U becomes I

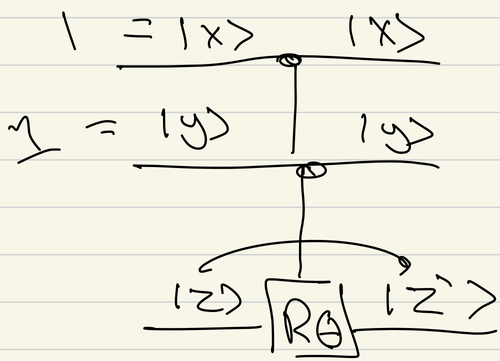
$$U = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

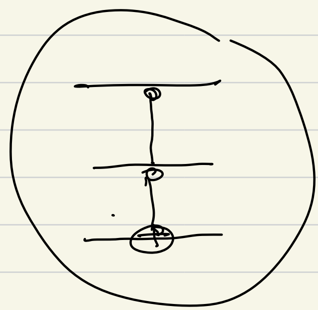
$$^cU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{bmatrix}$$

* Set of universal quantum gates

$$\{H, CNOT, R(\cos^{-1}(\tfrac{3}{5}))\}$$

* Deutsch gate





- Toffoli gate with $R(\theta)$ to change $|z\rangle$

No-Cloning Theorem

No-Hiding