Lecture 10

5/10/20

Ex 5.1.1

$$V = \begin{bmatrix} 5+3i \\ 6i \end{bmatrix}$$

$$|V| = \sqrt{70}$$

$$\frac{V}{\sqrt{70}} = \begin{bmatrix} \frac{5+3i}{\sqrt{70}} \\ \frac{6i}{\sqrt{70}} \end{bmatrix} = \frac{5+3i}{\sqrt{70}}|0\rangle + \frac{6i}{\sqrt{70}}|1\rangle$$

$$P(0) = \frac{34}{70} \quad , \quad P(1) = \frac{36}{70}$$

* $1/\sqrt{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1/\sqrt{2}|0\rangle + 1/\sqrt{2}|1\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$
* $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = 1/\sqrt{2}|0\rangle - 1/\sqrt{2}|1\rangle = \frac{|0\rangle-|1\rangle}{\sqrt{2}}$

$$\frac{|0\rangle+|1\rangle}{\sqrt{2}} = \frac{|1\rangle+|0\rangle}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\frac{|0\rangle-|1\rangle}{\sqrt{2}} \neq \frac{|1\rangle-|0\rangle}{\sqrt{2}}$$

$2^8 = 256$ possib vals

- A byte is 8 bits

$B = 01101011$

$$\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}$$

$$|0\rangle \otimes |1\rangle \otimes |0\rangle \otimes \cdots \otimes |1\rangle$$

$$\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2 = (\mathbb{C}^2)^{\otimes 8}$$

256-by-1 $\mathbb{C}^{256}$ $\quad 2^8 = 256$

qubyte

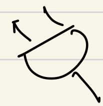
$$\begin{array}{l} 00000000 \\ 00000001 \\ \vdots \\ 01101011 \\ \\ 11111111 \end{array} \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} = B \qquad \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ \vdots \\ c_{255} \end{bmatrix} \begin{array}{l} -i/\sqrt{2} \\ i/\sqrt{2} \\ 1/\sqrt{2} \\ 1/n \end{array}$$

$$\sum_{i=0}^{255} |c_i|^2 = 1$$

* A qubit pair can be written

$$|0\rangle \otimes |1\rangle \text{ or } |0 \otimes 1\rangle \text{ or } |0\rangle|1\rangle \text{ or } |01\rangle$$

$$|01\rangle \quad \begin{matrix} 00 \\ 01 \\ 10 \\ 11 \end{matrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$



Ex

$$\frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{3}}|00\rangle - \frac{1}{\sqrt{3}}|10\rangle + \frac{1}{\sqrt{3}}|11\rangle$$

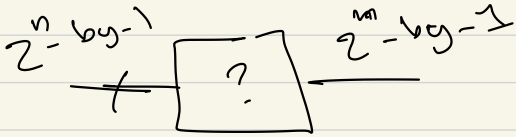
* The general state of two-qubit system can be written as

$$|\varphi\rangle = c_{0,0}|00\rangle + c_{0,1}|01\rangle + c_{1,0}|10\rangle + c_{1,1}|11\rangle$$

$$|0 \otimes 1\rangle \overset{?}{\neq} |1 \otimes 0\rangle$$

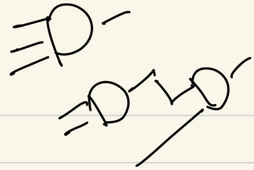
Sec 5.2 Classical gates

* Gates are ways to manipulate bits
* It takes bits as input, and output other bits
* A gate can take 2^n-by-1 matrix and produces a 2^m-by-1 matrix
*

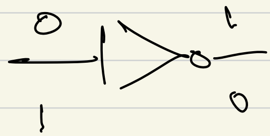


* A gate can by 2^m-by-2^n

$$\underbrace{(2^m\text{-by-}2^n)}_{\text{gate}}(2^n\text{-by-}1) = (2^m\text{-by-}1)$$



* NOT gate



A	O
0	1
1	0

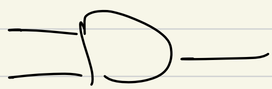
Not of $|0\rangle$ is $|1\rangle$
Not of $|1\rangle$ is $|0\rangle$

$$NOT = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{|0\rangle} = \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{|1\rangle}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

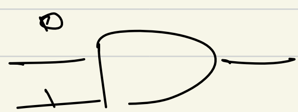
· AND gate



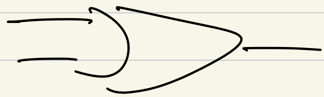
A B	O
0 0	0
0 1	0
1 0	0
1 1	1

$$AND = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 00 \\ 01 \\ 10 \\ 11 \end{matrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$



* OR gate

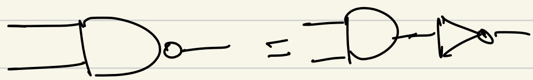


A	B	O
0	0	0
0	1	1
1	0	1
1	1	1

$$OR = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\overline{AB}$$

* NAND gate



$$NAND = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

A	B	O
0	0	1
0	1	1
1	0	1
1	1	0

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* NAND gate is a universal gate