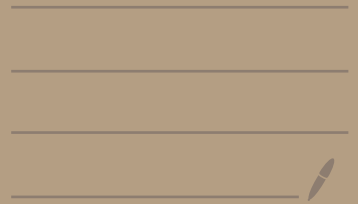


COE 466: Lecture 2

Review of Complex numbers



Lecture 2

9/2/2020

Objectives:

- Review Complex numbers
 - Review Complex Vector space
-

* Number sets

- Set of true numbers

$$\mathbb{P} = \{1, 2, 3, \dots\}$$

- Set of natural numbers

$$\mathbb{N} = \{0, 1, 2, \dots\}$$

- Set of integers (whole numbers)

$$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$$

- Set of rational numbers

$$\mathbb{Q} = \left\{ \frac{m}{n} \mid m \in \mathbb{Z}, n \in \mathbb{P} \right\}$$

- Set of real numbers

$$\mathbb{R} = \mathbb{Q} \cup \{\dots, \sqrt{2}, \dots, e, \dots, \pi\}$$

Example 1:

a. Solve $x^2 - 1 = 0$

$$x^2 = 1 \Rightarrow x = \pm 1$$

b. Solve $x^2 + 1 = 0$

$$x^2 = -1 \Rightarrow x = \pm \sqrt{-1}$$

i

* Imaginary number $i = \sqrt{-1}$

Example 2:

$$i^2 = -1$$

$$i^3 = -i = i \cdot i^2 = -i$$

$$i^4 = i \times i^3 = -i^2 = 1$$

$$i^5 = i \cdot i^4 = i$$

\vdots

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Def 1: A complex number is an expression

$$c = \underbrace{(a)}_{\substack{\text{real} \\ \text{part}}} + \underbrace{(b)i}_{\substack{\text{imaginary} \\ \text{part}}}$$

$c \in \mathbb{C}$ set of complex numbers

Example 3:

$$\text{let } c_1 = 3 - i, c_2 = 1 + 4i$$

a. Compute $c_1 + c_2$?

$$\begin{aligned} c_1 + c_2 &= (3 - i) + (1 + 4i) \\ &= 4 + 3i \end{aligned}$$

b. Compute $c_1 \times c_2$?

$$\begin{aligned} c_1 \times c_2 &= (3 - i)(1 + 4i) \\ &= 3 + 12i - i - 4 \\ &= 7 + 11i \end{aligned}$$

* Another notation for Complex number

$$c \mapsto (a, b)$$

$$a \mapsto (a, 0)$$

$$i \mapsto (0, 1)$$

* Let $c_1 = (a_1, b_1)$, $c_2 = (a_2, b_2)$

$$c_1 + c_2 = (a_1 + a_2, b_1 + b_2)$$

$$c_1 \times c_2 = (a_1 a_2 - b_1 b_2, a_1 b_2 + a_2 b_1)$$

Verify!

Example 4

$$c_1 = (3, -1), \quad c_2 = (1, 4)$$

$$c_1 + c_2 = (4, 3)$$

$$\begin{aligned} c_1 \times c_2 &= (3 + 4, 12 + (-1)) \\ &= (7, 11) \end{aligned}$$

* Complex number properties

- Commutative $c_1 + c_2 = c_2 + c_1$
 $c_1 \times c_2 = c_2 \times c_1$

- Associative

$$(c_1 + c_2) + c_3 = c_1 + (c_2 + c_3)$$

$$(c_1 \times c_2) \times c_3 = c_1 \times (c_2 \times c_3)$$

- multiplication distribute over
addition

$$c_1 \times (c_2 + c_3) = (c_1 \times c_2) + (c_1 \times c_3)$$

Verify!

* Subtraction

$$c_1 - c_2 = (a_1, b_1) - (a_2, b_2) \\ = (a_1 - a_2, b_1 - b_2)$$

* Division

$$\frac{(a_1, b_1)}{(a_2, b_2)} = (x, y) \Rightarrow$$

$$(a_1, b_1) = (x, y) (a_2, b_2)$$

So, we end up with $(a_2x - b_2y, b_2x + a_2y)$

$$a_2 a_1 = (a_2x - b_2y) a_2 - \textcircled{1}$$

$$b_2 b_1 = (b_2x + a_2y) b_2 - \textcircled{2}$$

$$a_1 a_2 + b_1 b_2 = a_2^2 x + b_2^2 x$$

$$x = \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2}$$

$$y = \frac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2}$$

$$\frac{(a_1, b_1)}{(a_2, b_2)} = \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2}, \frac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2}$$

Example 5:

$$\frac{(-2, 1)}{(\cancel{1}, 2)} = \frac{-2 + \overset{0}{\cancel{2}}}{5}, \quad \frac{1 - \overset{5}{\cancel{4}}}{5} \quad \underline{1}$$

$$x=0, \quad y=1$$

$$c = \hat{z}$$

* In \mathbb{R} , absolute value is

$$|a| = +\sqrt{a^2}$$

$$|5| = +\sqrt{25} = 5$$

$$|-3| = +\sqrt{9} = 3$$

* Modulus of Complex number is

$$|c| = |a+bi| = +\sqrt{a^2+b^2}$$

$$|c|^2 = a^2+b^2$$