COE 466: Lecture 2

Review of Complex numbers

Lecture 2 9/2/2020

Objectives:

- Review Complex numbers
- Review Complex Vector space

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* Number sets

- Set of +ve numbers

  $\mathbb{P} = \{1, 2, 3, ...\}$

- Set of natural numbers

  $\mathbb{N} = \{0, 1, 2, ...\}$

- Set of integers (whole numbers

  $\mathbb{Z} = \{..., -2, -1, 0, 1, 2, ...\}$

- Set of rational numbers

  $\mathbb{Q} = \{\frac{m}{n} \mid m \in \mathbb{Z}, n \in \mathbb{P}\}$

- Set of real numbers

  $\mathbb{R} = \mathbb{Q} \cup \{..., \sqrt{2}, ..., e, ..\pi\}$

Example 1:

a- Solve $x^2 - 1 = 0$

$$x^2 = 1 \Rightarrow x = \pm 1$$

b- Solve $x^2 + 1 = 0$

$$x^2 = -1 \Rightarrow x = \pm \sqrt{-1}$$

i

* Imaginary number $i = \sqrt{-1}$

Example 2:

$$i^2 = -1$$

$$i^3 = -i = i\,i^2 = -i$$

$$i^4 = i \times i^3 = -i^2 = 1$$

$$i^5 = i\,i^4 = i$$

$\vdots$

Def 1 : A complex number is an expression

$$c = a + b i$$

real part   imaginary part

$c \in \mathbb{C}$ set of complex number

Example 3:

let $c_1 = 3 - i$, $c_2 = 1 + 4i$

a_ Compute $c_1 + c_2$?

$$c_1 + c_2 = (3 - i) + (1 + 4i)$$
$$= 4 + 3i$$

b_ Compute $c_1 \times c_2$?

$$c_1 \times c_2 = (3 - i)(1 + 4i)$$
$$3 + 12i - i + 4$$
$$= 7 + 11i$$

* Another notation for Complex number

$$c \mapsto (a, b)$$
$$a \mapsto (a, 0)$$
$$i \mapsto (0, 1)$$

* Let $c_1 = (a_1, b_1)$, $c_2 = (a_2, b_2)$

$$c_1 + c_2 = (a_1 + a_2, b_1 + b_2)$$
$$c_1 \times c_2 = (a_1 a_2 - b_1 b_2, a_1 b_2 + a_2 b_1)$$

Verify!

<u>Example 4</u>

$$c_1 = (3, -1), \quad c_2 = (1, 4)$$
$$c_1 + c_2 = (4, 3)$$

$$c_1 \times c_2 = (3 + 4, 12 + (-1))$$
$$= (7, 11)$$

* Complex number properties
  - Commutative $c_1 + c_2 = c_2 + c_1$
    $c_1 \times c_2 = c_2 \times c_1$
  - Associative
    $(c_1 + c_2) + c_3 = c_1 + (c_2 + c_3)$
    $(c_1 \times c_2) \times c_3 = c_1 \times (c_2 \times c_3)$
  - multiblication distribute over addition
    $c_1 \times (c_2 + c_3) = (c_1 \times c_2) + (c_1 \times c_3)$
    Verify!

* Subtraction
  $c_1 - c_2 = (a_1, b_1) - (a_2, b_2)$
  $= (a_1 - a_2, b_1 - b_2)$

* Division

$$\frac{(a_1, b_1)}{(a_2, b_2)} = (x, y) \Rightarrow$$

$$(a_1, b_1) = (x, y)(a_2, b_2)$$

$$(a_2x - b_2y, b_2x + a_2y)$$

So, we end up with

$$a_2 a_1 = (a_2 x - \cancel{b_2 y}) a_2 \quad - (1)$$

$$b_2 b_1 = (b_2 x + \cancel{a_2 y}) b_2 \quad - (2)$$

$$a_1 a_2 + b_1 b_2 = a_2^2 x + b_2^2 x$$

$$x = \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2}$$

$$y = \frac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2}$$

$$\frac{(a_1, b_1)}{(a_2, b_2)} = \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2}, \frac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2}$$

Example 5:

$$\frac{(-2,1)}{(\cancel{1},2)} = \frac{\overset{0}{\cancel{-2+2}}}{\cancel{5}} \; , \; \frac{\overset{5}{\cancel{1-(-4)}}}{5} \quad 1$$

$$x = 0 \; , \; y = 1$$

$$c = i$$

* In $\mathbb{R}$, absolute value :

$$|a| = +\sqrt{a^2}$$
$$|5| = +\sqrt{25} = 5$$
$$|-3| = +\sqrt{9} = 3$$

* Modulus of Complex number is

$$|c| = |a+bi| = +\sqrt{a^2+b^2}$$
$$|c|^2 = a^2+b^2$$