King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Computer Engineering Department
COE 466: Quantum Architecture and Algorithms

Problem Set 1

Due date: Monday 21-9-2020 (Before the class)

Problem Sets

1. Prove that
   $|c_1 + c_2| \leq |c_1| + |c_2|$ where $c_1, c_2 \in \mathbb{C}$.
   (hint: square both sides).

2. Show that conjugation respects addition, i.e.,
   $\overline{c_1} + \overline{c_2} = \overline{c_1 + c_2}$

3. Find the transpose, conjugate, and adjoint of the following matrix

$$\begin{bmatrix} 6-3i & 2+12i & -19i \\ 0 & 5+2.1i & 17 \\ 1 & 2+5i & 3-4.5i \end{bmatrix}$$

4. Let $c_1 = 2i, c_2 = 1 + 2i$, and $A = \begin{bmatrix} 1-i & 3 \\ 2+2i & 4+i \end{bmatrix}$. Verify the following property. $c_1 \cdot (c_2 \cdot A) = (c_1 \times c_2) \cdot A$

5. Let $A = \begin{bmatrix} 3+2i & 0 & 5-6i \\ 1 & 4+2i & i \\ 4-i & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2-i & 6-4i \\ 0 & 4+5i & 2 \\ 6-4i & 2+7i & 0 \end{bmatrix}$.
   Show that $(A * B)^\dagger = B^\dagger * A^\dagger$

6. Show that the set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ -4 \end{bmatrix} \right\}$$

   is not linearly independent.

7. Calculate the norm of $\begin{bmatrix} 4+3i \\ 6-4i \\ 12-7i \\ 13i \end{bmatrix}$

8. Let $V_1 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $V_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$. Calculate the distance between these two vectors.

9. Show that a matrix A is hermitian if and only if $A^T = \overline{A}$

10. Show that the matrix

$$\begin{bmatrix} \frac{1+i}{2} & \frac{i}{\sqrt{3}} & \frac{3+i}{2\sqrt{15}} \\ \frac{-1}{2} & \frac{1}{\sqrt{3}} & \frac{4+3i}{2\sqrt{15}} \\ \frac{1}{2} & \frac{-i}{\sqrt{3}} & \frac{5i}{2\sqrt{15}} \end{bmatrix}$$

is a unitary matrix.

11. Calculate the tensor product $\begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix} \otimes \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

12. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 6 & 5 \\ 3 & 2 \end{bmatrix}$. Calculate $A \otimes (B \otimes C)$ and $(A \otimes B) \otimes C$ and show that they are equal.