



**King Fahd University of Petroleum and Minerals  
Department of Computer Engineering**

**DIGITAL LOGIC DESIGN COE 202**

**Homework 1, October 18, 2008**

<b>Problems</b>	<b>Grading</b>
1	
2	
3	
4	
5	
<b>TOTAL</b>	

**Student Name:.....**

**Student ID:.....**

## Digital Computer and Information

Answer each of the following questions and show all your steps

- What is the decimal equivalent of the largest binary integer that can be obtained with (a) 12 bits, and (2) 24 bits.
- Q-2: Convert the following binary numbers to decimal representation  $(1001101)_2$ ,  $(1010011.101)_2$ , and  $(10101110.1001)_2$ .
- Each of the following numbers has a different base  $(11100111)_2$ ,  $(22120)_3$ ,  $(3113)_4$ ,  $(4110)_5$ ,  $(530)_8$ ,  $(343)_8$ . Convert the numbers from the given basis (radix) to decimal. Which of the five numbers have the same value in decimal?
- Convert the following numbers from the given basis (radix)
  - $(369.3125)_{10}$  to binary, octal and hexadecimal
  - $(10111101.101)_2$  to decimal, octal and hexadecimal
  - $(326.5)_8$  to decimal, binary and hexadecimal
  - $(F3C7.A)_{16}$  to decimal, binary and octal

List the results of above conversion in a table for each of the above numbers.

- Suppose we have  $(BEE)_r = (2699)_{10}$  and  $(365)_r = (194)_{10}$ , where r is the radix (base) and B=11 and E=14 in decimal. Determine the value of the radix r for each equation?

**Solution:**

### **1. Question-1:**

$$\begin{aligned} \text{(a)} \quad 12 \text{ bits} &\Rightarrow 2^{12} - 1 = (4,095)_{10} \\ \text{(b)} \quad 24 \text{ bits} &\Rightarrow 2^{24} - 1 = (16,777,215)_{10} \end{aligned}$$

### **2. Question-2:**

$$\begin{array}{llll} \text{(a)} \quad (1001101)_2 & = 2^6 + 2^3 + 2^2 + 2^0 & & = (77)_{10} \\ \text{(b)} \quad (1010011.101)_2 & = 2^6 + 2^4 + 2^1 + 2^0 + 2^{-1} + 2^{-3} & & = (83.625)_{10} \\ \text{(c)} \quad (10101110.1001)_2 & = 2^7 + 2^5 + 2^3 + 2^2 + 2^1 + 2^{-1} + 2^{-4} & & = (174.5625)_{10} \end{array}$$

### **3. Question-3:**

$$\begin{array}{llll} \text{(a)} \quad (11100111)_2 & = 2^7 + 2^6 + 2^5 + 2^2 + 2^1 + 2^0 = 128 + 64 + 32 + 4 + 2 + 1 & & = (231)_{10} \\ \text{(b)} \quad (22120)_3 & = 2 * 3^4 + 2 * 3^3 + 1 * 3^2 + 2 * 3^1 = 162 + 54 + 9 + 6 & & = (231)_{10} \\ \text{(c)} \quad (3113)_4 & = 3 * 4^3 + 1 * 4^2 + 1 * 4^1 + 3 * 4^0 = 192 + 16 + 4 + 3 & & = (215)_{10} \\ \text{(d)} \quad (4110)_5 & = 4 * 5^3 + 1 * 5^2 + 1 * 5^1 = 500 + 25 + 5 & & = (530)_{10} \\ \text{(e)} \quad (343)_8 & = 3 * 8^2 + 4 * 8^1 + 3 * 8^0 = 192 + 32 + 3 & & = (227)_{10} \end{array}$$

### **4. Question-4:**

(a)  $(369.3125)_{10}$

$$\begin{array}{r}
 369 \left| \begin{array}{l} \bullet \\ 184 \\ 92 \\ 46 \\ 23 \\ 11 \end{array} \right. \\
 184 \left| \begin{array}{l} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{array} \right. \\
 92 \left| \begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{array} \right. \\
 46 \left| \begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{array} \right. \\
 23 \left| \begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{array} \right. \\
 11 \left| \begin{array}{l} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{array} \right. 
 \end{array}
 \begin{array}{r}
 0.3125 \times 2 = 0.625 \quad \bullet \\
 0.625 \times 2 = 1.25 \\
 0.25 \times 2 = 0.5 \\
 0.5 \times 2 = 1.0
 \end{array}$$

5	1
2	1
1	0
0	1

$$\begin{aligned}
 (369.3125)_{10} &= (101110001.0101)_2 \\
 &= (\textcolor{red}{101110001.0101}\textcolor{red}{00})_2 = (561.24)_8 \\
 &= (\textcolor{red}{000101110001.0101})_2 = (171.5)_{16}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad (10111101.101)_2 &= 2^7 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0 + 2^{-1} + 2^{-3} \\
 &= 128 + 32 + 16 + 8 + 4 + 1 + 0.5 + 0.125 \\
 &= (189.625)_{10} \\
 &= (\textcolor{red}{010111101.101})_2 = (275.5)_8 \\
 &= (\textcolor{red}{10111101.1010})_2 = (\text{BD.A})_{16}
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad (326.5)_8 &= 3 * 8^2 + 2 * 8^1 + 6 * 8^0 + 5 * 8^{-1} \\
 &= 192 + 16 + 6 + 0.625 \\
 &= (214.625)_{10}
 \end{aligned}$$

$$(\textcolor{red}{326.5})_8 = (\textcolor{red}{011010110.101})_2 = (11010110.101)_2$$

$$(326.5)_8 = (\textcolor{red}{11010110.1010})_2 = (\text{D6.A})_{16}$$

$$\begin{aligned}
 (d) \quad (\text{F3C7.A})_{16} &= 15 * 16^3 + 3 * 16^2 + 12 * 16^1 + 7 * 16^0 + 10 * 16^{-1} \\
 &= 61440 + 768 + 192 + 7 + 0.625 \\
 &= (62407.625)_{10}
 \end{aligned}$$

$$\begin{aligned}
 (\text{F3C7.A})_{16} &= (\textcolor{red}{1111001110001111.1010})_2 = (1111001111000111.101)_2 \\
 (\text{F3C7.A})_{16} &= (\textcolor{red}{001111001111000111.101})_2 = (171707.5)_8
 \end{aligned}$$

Decimal	Binary	Octal	Hexadecimal
369.3125	101110001.0101	561.24	171.5
189.625	10111101.101	275.5	BD.A
214.625	11010110.101	326.5	D6.A
62407.625	1111001111000111.101	171707.5	F3C7.A

## 5. Question-5:

$$\begin{aligned}
 (a) \quad (\text{BEE})_r = (2699)_{10} &\Rightarrow 11 * r^2 + 14 * r^1 + 14 * r^0 = (2699)_{10} \\
 &\Rightarrow 11 * r^2 + 14 * r + 14 = (2699)_{10} \quad (\text{note that } r \text{ must be } \geq 15) \\
 &\Rightarrow 11 * r^2 + 14 * r - 2685 = 0 \\
 &\Rightarrow (11r + 179)(r - 15) = 0 \\
 &\Rightarrow r = -16.27 \quad \text{or} \quad r = 15 \\
 &\Rightarrow r = 15
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad (365)_r = (194)_{10} &\Rightarrow 3 * r^2 + 6 * r^1 + 5 * r^0 = (194)_{10} \\
 &\Rightarrow 3 * r^2 + 6 * r + 5 = (194)_{10} \quad (\text{note that } r \text{ must be } \geq 7) \\
 &\Rightarrow 3 * r^2 + 6 * r - 189 = 0 \\
 &\Rightarrow r^2 + 2 * r - 63 = 0 \\
 &\Rightarrow (r + 9)(r - 7) = 0 \\
 &\Rightarrow r = -9 \quad \text{or} \quad r = 7 \\
 &\Rightarrow r = 7
 \end{aligned}$$