King Fahd University of Petroleum and Minerals
Department of Computer Engineering

DIGITAL LOGIC DESIGN COE 202
Homework 2, November 16, 2008

Problems	Grading
1	
2	
3	
4	
5	
TOTAL	

Student Name:……

Student ID:……

1. Question-1: Solve each of the following three problems:

    1. Using Algebraic manipulation prove the following algebraic identities:

        a. X'Y'+X'Y+XY = X'+Y
        b. A'B+B'C'+AB+B'C= 1
        c. Y+X'Z+XY'=X+Y+Z
        d. X'Y'+Y'Z+XZ+XY+YZ'=X'Y'+XZ+YZ'

    2. Convert the following expressions into (1) Sum-of-Product, and (2) Product-of-Sum forms:

        a. (AB+C)(B+C'D)
        b. X'+X(X+Y')(Y+Z')
        c. (A+BC'+CD)(B'+EF)

    3. Draw the logic diagram for the following logical expressions:

        a. WX'Y'+W'Z+YZ
        b. A(BD'+B'D)+D(BC+B'C')
        c. WY'(X+Z)+X'Z(W+Y)+WX'(Y+Z)

2. Question-2: Write the *sum-of-minterms* and *product-of-maxterms* expressions for both the *true* and the *complement* form of the following Boolean expressions:

    a. $X\bar{Z} + Y\bar{Z} + XYZ$
    b. $\bar{A}\bar{B} + A\bar{C}D + \bar{B}C + \bar{A}B\bar{C}\bar{D}$

3. Question-3: For the given truth table and using a K-map:

    a. Find all ***prime implicants*** and ***essential prime implicants*** of *F*.
    b. Write an optimized SOP **<u>and</u>** POS expressions for the function *F*.

A	*B*	*C*	*D*	*F*
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

4. For the Boolean function E and F, as given in the following truth table:

X	Y	Z	E	F
0	0	0	1	0
0	0	1	1	0
0	1	0	1	1
0	1	1	0	0
1	0	0	1	0
1	0	1	0	0
1	1	0	0	1
1	1	1	0	1

a. List the minterms and the maxterms of each function.

b. List the minterms of E' and F'.

c. List the minterms of E + F and E . F.

d. Express E and F in sum-of-minterms and product-of-maxterms algebraic form.

Solution:

Question-1, Problem I:

(a) $\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X}(\bar{Y} + Y) + XY$

$= \bar{X} + XY$

$= (\bar{X} + X)(\bar{X} + Y)$

$= \bar{X} + Y$

(b) $\bar{A}B + \bar{B}\bar{C} + AB + \bar{B}C = (\bar{A} + A)B + \bar{B}(\bar{C} + C)$

$= B + \bar{B}$

$= 1$

(c) $Y + \bar{X}Z + X\bar{Y} = \bar{X}Z + Y + X\bar{Y}$

$= \bar{X}Z + (Y + X)(Y + \bar{Y})$

$= \bar{X}Z + Y + X$

$= \bar{X}Z + X + Y$

$= (\bar{X} + X)(Z + X) + Y$

$= X + Y + Z$

(d)

$$\begin{aligned}
\bar{X}\bar{Y} + Y\bar{Z} + XZ + XY + Y\bar{Z} &= \bar{X}\bar{Y} + Y\bar{Z}(X + \bar{X}) + XZ + XY + Y\bar{Z} \\
&= \bar{X}\bar{Y} + XY\bar{Z} + \bar{X}Y\bar{Z} + XZ + XY + Y\bar{Z} \\
&= \bar{X}\bar{Y}(1 + Z) + XY\bar{Z} + XZ + XY + Y\bar{Z} \\
&= \bar{X}\bar{Y} + XZ(\bar{Y} + 1) + XY + Y\bar{Z} \\
&= \bar{X}\bar{Y} + XZ + XY(Z + \bar{Z}) + Y\bar{Z} \\
&= \bar{X}\bar{Y} + XZ + XYZ + XY\bar{Z} + Y\bar{Z} \\
&= \bar{X}\bar{Y} + XZ(1 + Y) + Y\bar{Z}(X + 1) \\
&= \bar{X}\bar{Y} + XZ + Y\bar{Z}
\end{aligned}$$

Question-1, Problem II:

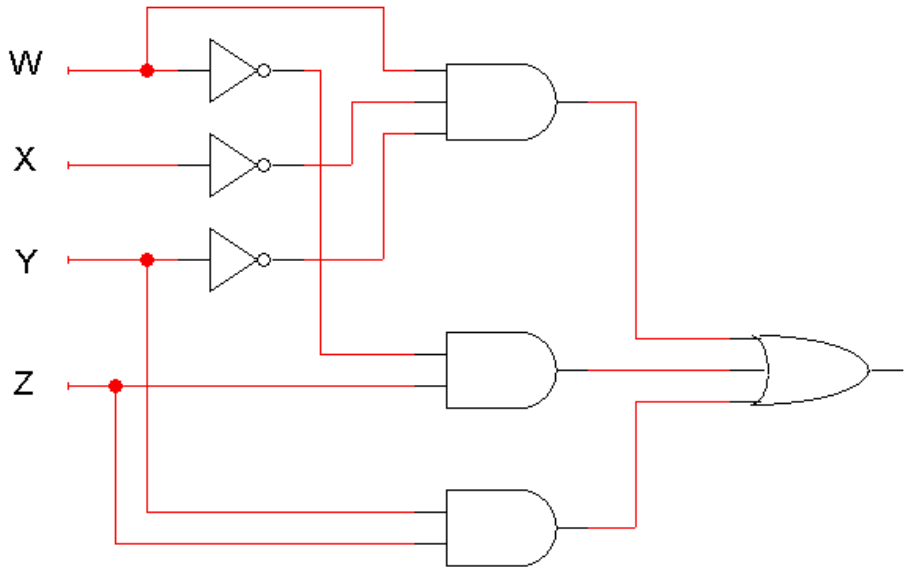
(a)
$$\begin{aligned}
(AB + C)(B + \bar{C}D) &= ABB + AB\bar{C}D + BC + C\bar{C}D \\
&= AB + AB\bar{C}D + BC \text{ (SOP)}
\end{aligned}$$
$$(AB + C)(B + \bar{C}D) = (A + C)(B + C)(B + \bar{C})(B + D) \text{ (POS)}$$

(b)
$$\begin{aligned}
\bar{X} + X(X + \bar{Y})(Y + \bar{Z}) &= \bar{X} + (XX + X\bar{Y})(Y + \bar{Z}) \\
&= \bar{X} + (X + X\bar{Y})(Y + \bar{Z}) \\
&= \bar{X} + XY + XY\bar{Y} + X\bar{Z} + XY\bar{Z} \\
&= \bar{X} + XY + X\bar{Z} + XY\bar{Z} \text{ (SOP)} \\
&= \bar{X} + Y + \bar{Z} \text{ (simplified SOP)}
\end{aligned}$$
$$\begin{aligned}
\bar{X} + X(X + \bar{Y})(Y + \bar{Z}) &= (\bar{X} + X)(\bar{X} + X + \bar{Y})(\bar{X} + Y + \bar{Z}) \\
&= (\bar{X} + Y + \bar{Z}) \text{ (POS)}
\end{aligned}$$

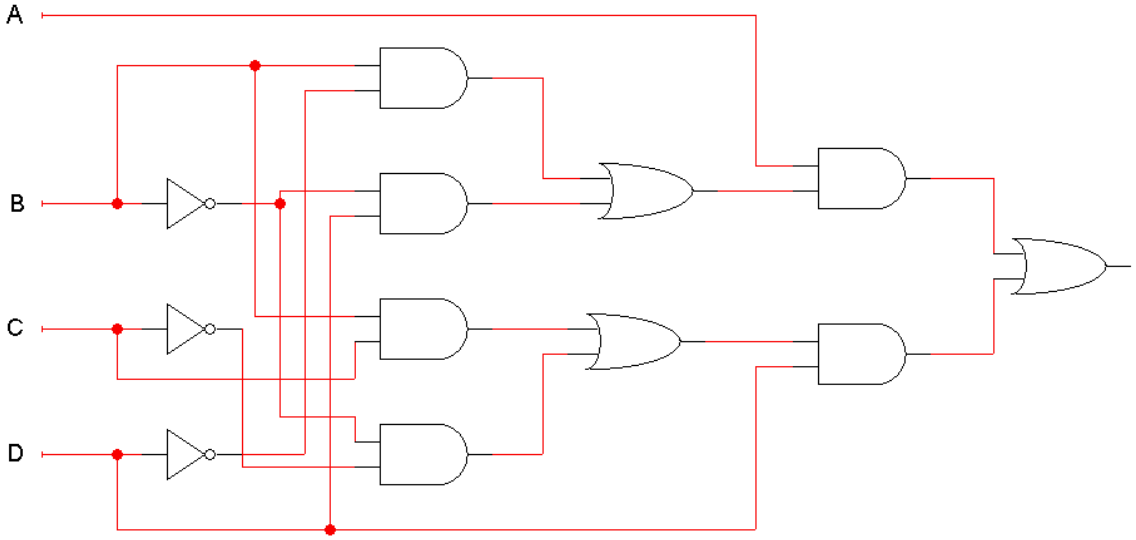
(c)
$$\begin{aligned}
(A + B\bar{C} + CD)(\bar{B} + EF) &= A\bar{B} + B\bar{C}\bar{B} + CD\bar{B} + AEF + B\bar{C}EF + CDEF \\
&= A\bar{B} + \bar{B}CD + AEF + B\bar{C}EF + CDEF \text{ (SOP)}
\end{aligned}$$
$$(A + B\bar{C} + CD)(\bar{B} + EF) = (A + B)(A + \bar{C})(A + C)(A + D)(\bar{B} + E)(\bar{B} + F) \text{ (POS)}$$

Question-1, Problem III:

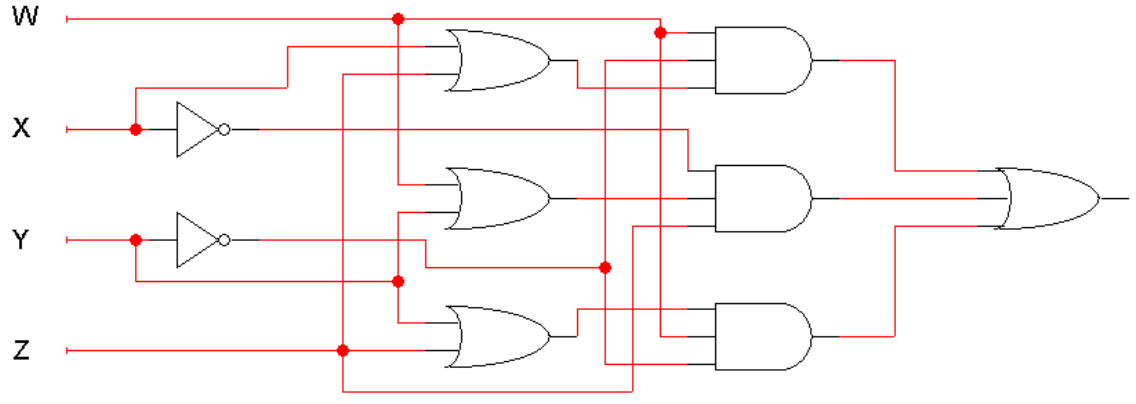
(a) $W\bar{X}\bar{Y} + W\bar{Z} + YZ$



(b) $A(B\bar{D}+\bar{B}D)+D(BC+\bar{B}\bar{C})$



(c) $W\bar{Y}(X+Z)+\bar{X}Z(W+Y)+W\bar{X}(Y+Z)$



Question-2

Write the *sum-of-minterms* **and** *product-of-maxterms* expressions **for both** the *true* and the *complement* form of the following Boolean expressions:

- $\bar{X}\bar{Z} + Y\bar{Z} + XYZ$

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$F(X,Y,Z) = \sum(0,2,6,7) = \prod(1,3,4,5)$$
$$\bar{F}(X,Y,Z) = \sum(1,3,4,5) = \prod(0,2,6,7)$$

- $\bar{A}\bar{B} + A\bar{C}D + \bar{B}C + \bar{A}B\bar{C}\bar{D}$

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

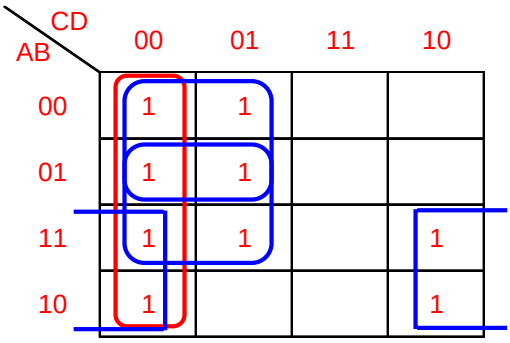
$$F(A,B,C,D) = \sum(0,1,2,3,4,9,10,11,13) = \prod(5,6,7,8,12,14,15)$$
$$\bar{F}(A,B,C,D) = \sum(5,6,7,8,12,14,15) = \prod(0,1,2,3,4,9,10,11,13)$$

Question-3

For the given truth table and using a K-map:

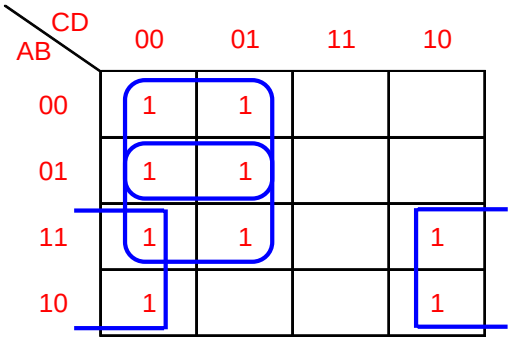
- (*5 points*) Find all *prime implicants* and ***essential*** *prime implicants* of *F*.
- (*10 points*) Write an optimized SOP **and** POS expressions for the function *F*.

A	*B*	*C*	*D*	*F*
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

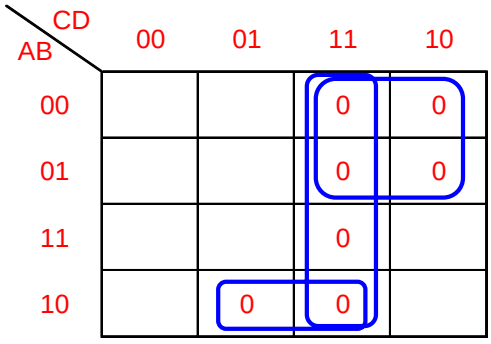


PIs (both blue and red): $A\bar{D}, \bar{A}\bar{C}, B\bar{C}, \bar{C}\bar{D}$

EPIs (only blue): $A\bar{D}, \bar{A}\bar{C}, B\bar{C}$



$F(A,B,C,D) = SOP = A\bar{D} + \bar{A}\bar{C} + B\bar{C}$



$F(A,B,C,D) = POS = (\bar{A} + B + \bar{D}) \cdot (\bar{C} + \bar{D}) \cdot (A + \bar{C})$

4. Question 4

a. $E = \Sigma (0, 1, 2, 4) = \Pi (3, 5, 6, 7)$; $F = \Sigma (2, 6, 7) = \Pi (0, 1, 3, 4, 5)$

b. $E' = \Sigma (3, 5, 6, 7) = xy'z' + xy'z + xyz' + xyz$
$F' = \Sigma (0, 1, 3, 4, 5) = x'y'z' + x'y'z + x'yz + xy'z' + xy'z$

c. $E+F= \Sigma (0, 1, 2, 4, 6, 7)$; $E.F= \Sigma (0)$

d. $E = x'y'z' + x'y'z + x'yz' + xy'z' = (x + y' + z')\ (x' + y + z')\ (x' + y' + z) (x' + y' + z'); \quad F = x'yz' + xyz' + xyz = (x + y + z) (x + y + z') (x + y' + z') (x' + y + z) (x' + y + z')$