Question 1: <u>Fill in the Spaces</u>: (Show all work needed to obtain your answer) **(13 points)**

a. To represent the decimal number 65 in binary we need ________ (how many) bits. **[1 point]**

Solution:

$$Required\ bits = \lfloor log_2 65 \rfloor + 1 = \lfloor 6.02 \rfloor + 1 = \mathbf{7}\ bits$$

b. $(1324)_5$ = (_____________)$_{10}$ **[2 points]**

Solution:

$$(1324)_5 = 1\times5^3 + 3\times5^2 + 2\times5^1 + 4\times5^0 = 125 + 75 + 10 + 4 = (\mathbf{214})_{10}$$

c. A communication system uses a 1-bit parity scheme for error detection. The receiver receives a byte represented in hexadecimal as **D3** without error. The parity scheme used is __________ (even/odd) parity. **[1 point]**

Solution:

$$(D3)_{16} = (1101\ 0011)_2$$
$$count\ of\ 1's = 5$$
$$5\ mod\ 2 = 1, \quad 0{:}\ even, 1{:}\ odd$$
$$\boldsymbol{odd}$$

d. For 5 variables (A, B, C, D, E), m_{13} = ______________ (algebraic expression), while the algebraic expression $(\overline{A}+B+\overline{C}+\overline{D}+E)$ represents the maxterm M? _________. **[2 points]**

Solution:

$$(13)_{10} = (01101)_2$$
$$m_{13} = \boldsymbol{\bar{A}BC\bar{D}E}$$

$$(\bar{A} + B + \bar{C} + \bar{D} + E) \rightarrow (10110)_2 = (22)_{10}$$
$$(\bar{A} + B + \bar{C} + \bar{D} + E)\ \text{represents the maxterm}\ \boldsymbol{M_{22}}$$

e. The canonical form (sum of minterms or product of maxterms) represents the most simplified form of a logic function _________(True/False). **[1 Point]**

Solution:

False

f. The number of minterms and maxterms in the function $F(A,B,C) = A + B + \overline{C}$ is __________ minterms and __________ maxterms. **[2 points]**

Solution:

$$F(A,B,C) = A + B + \bar{C} = A(B+\bar{B})(C+\bar{C}) + B(A+\bar{A})(C+\bar{C}) + \bar{C}(A+\bar{A})(B+\bar{B})$$
$$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \cancel{ABC} + \cancel{AB\bar{C}} + \bar{A}BC + \bar{A}B\bar{C} + \cancel{AB\bar{C}} + \cancel{A\bar{B}\bar{C}}$$
$$+ \bar{A}\cancel{B\bar{C}} + \bar{A}\bar{B}\bar{C} = \sum m(0,2,3,4,5,6,7) = \prod M(1)$$

7 minterms, **1** maxterm

g. Given the identity: $AB+\overline{A}C+BC=AB+\overline{A}C$, using the duality principle $(A+B)(\overline{A}+C)(B+C)=$____. The property/theorem is known as __________________ theorem. **[2 points]**

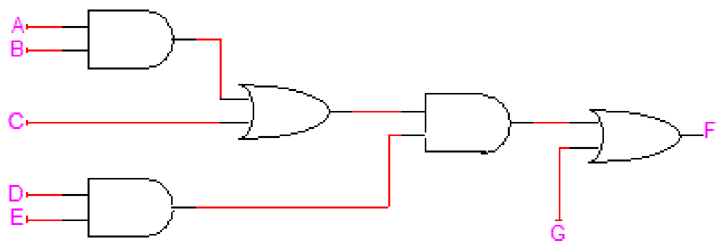
Solution:

$(\boldsymbol{A}+\boldsymbol{B})(\overline{\boldsymbol{A}}+\boldsymbol{C})$

consensus theorem

h. For the logic circuit shown below, assuming that all gates have the same propagation delay of **2 ns**, then the circuit takes __________ ns to produce the correct output.

[2 points]



Solution:

The worst-case propagation delay from input to output is the delay of the path containing the maximum number of logic gates, since all gates have the same propagation delay. This path is: AND-OR-AND-OR

The mentioned path contains 4 logic gates each with propagation delay of 2 ns. So, the circuit takes $4\times2=\mathbf{8}$ ns to produce the correct output.

Question 2: Perform the following conversions: (6 points)

i. $(110100.011)_2$ to decimal. **[1 point]**

$(110100.011)_2 = 4 + 16 + 32 + 4^{-1} + 8^{-1} = (52.375)_{10}$

ii. $(59.7)_{10}$ to binary (use up to 4 fractional bits accuracy). **[2 points]**

$(59.7)_{10} = (111011.1011)_2$

iii. $(651.13)_8$ to hexadecimal. **[2 points]**

$(651.13)_8 = (000110101001.00101100)_2 = (1A9.2C)_{16}$

Question 3: Without converting to other bases, find the result of the following arithmetic operations:

i. $(57.6)_{16} + (4E.7)_{16}$ **[2 points]**

$$\begin{array}{r} (57.6)_{16} \\ +\ (4E.7)_{16} \\ \hline (A5.D)_{16} \end{array}$$

ii. $(111)_2 \times (110)_2$ **[2 points]**

$$\begin{array}{r} (111)_2 \\ \times\ (110)_2 \\ \hline (\quad 000)_2 \\ (\ 111\ )_2 \\ (111\quad)_2 \\ \hline (101010)_2 \end{array}$$

iii. $(110100)_2 - (100101)_2$ **[2 points]**

$$\begin{array}{r} (110100)_2 \\ -\ (100101)_2 \\ \hline (001111)_2 \end{array}$$

Question 4:

a) List all the **Minterms** and **Maxterms** of the following Boolean function (using the **Σ** and **Π** notations):

$$f(x,y,z) = xy + (x'+z)(y+z')$$ **[4 points]**

Solution: Truth table for f

$x\ y\ z$	xy	$(x'+z)$	$(y+z')$	$(x'+z)(y+z')$	f
0 0 0	0	1	1	1	1
0 0 1	0	1	0	0	0
0 1 0	0	1	1	1	1
0 1 1	0	1	1	1	1
1 0 0	0	0	1	0	0
1 0 1	0	1	0	0	0
1 1 0	1	0	1	0	1
1 1 1	1	1	1	1	1

List of minterms = $\sum(0,2,3,6,7) = m_0 + m_2 + m_3 + m_6 + m_7$

List of Maxterms = $\prod(1,4,5) = M_1 \cdot M_4 \cdot M_5$

b) Given the following Boolean functions f and g: (6 points)

$f(x,y,z) = \sum(1,3,6)$

$g(x,y,z) = \sum(0,2,4,6,7)$

i) Write an **algebraic** expression for f as a sum-of-minterms (2 points)

ii) Write an **algebraic** expression for $(f' \cdot g)$ as a product-of-maxterms (4 points)

Solution:

i) $f(x,y,z) = \sum(1,3,6) = x'y'z + x'y\,z + x\,y\,z'$

ii) $f' = \sum(0,2,4,5,7)$

$f'.g = \sum(0,2,4,7) = \prod(1,3,5,6) = (x+y+z')(x+y'+z')(x'+y+z')(x'+y'+z)$

Question 5: Consider the following circuit with two outputs f and g.

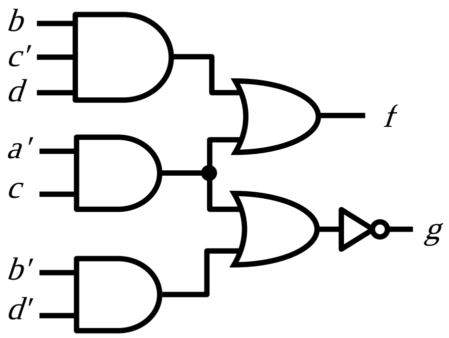
a) Write an expression for the output ***f*** as a **sum-of-products** (1 point)

b) Write an expression for the output ***g*** as a **product-of-sums** (2 points)

Solution:

$f = bc'd + a'c$

$g = (a'c + b'd')' = (a+c')(b+d)$



Question 6: Find the complement of each of the function below as they are (i.e. **do not change or simplify them first**): (4 points)

a) $F = (XY + Z) \cdot \overline{W} \cdot (E + \overline{D})$ **[2 Points]**

b) $G = A + D + B \cdot C \cdot \overline{E}$ **[2 Points]**

a) F' = (X' + Y')Z' + W + E'D
b) G'= A'D'(B' + C' + E)

Question 7: Using **Boolean Algebra** and showing all steps of your work: **[8 points]**

i. Proof that: $\overline{X(\overline{Y} + Z)} = \overline{X}\,\overline{Z} + \overline{X}\,\overline{Y} + XY\overline{Z} + \overline{X}YZ$ **[4 points]**

Solution:

X' + YZ' (LFS)
RHS = X'(Z' + Y~~Z~~) + X'Y' + XYZ'
= X'Z' + X'Y + X'Y' + XYZ'
= Z' (X'+ ~~X~~Y) + X'~~(Y + Y')~~
= X' + X'Z' + YZ'
= X'(~~1 + Z'~~) + YZ'
= X' + YZ' = LHS

ii. Simplify the following function to minimum number of literals in **SOP** form:

F (A,B,C,D) = ∑ m(8,10,12,14) **[4 points]**

Solution:
1st we express F algebraically
F = AB'C'D' + AB'CD' + ABC'D' + ABCD'
= AD'(B'C' + B'C + BC' + BC)
= AD'(B'(~~C'+C~~) + B(~~C'+C~~)) AD'(~~B'+B~~)
= AD'