## Homework \# 07 (due date \& time: Sunday 24/04/2016 during class period)

*** Show all your work. No credit will be given if work is not shown! ***

## Problem 1 (30 points):

(a) Evaluate the integral $\int_{0}^{\pi / 2}(5-3 \sin x) d x$ using:
i. ( $\mathbf{1 0}$ points) Single application of the trapezoidal rule.
ii. (10 points) Multiple-application trapezoidal rule $n=4$.
(b) (10 points) Estimate the true percent relative error $\varepsilon_{t}$ for each approximation.

Problem 2 (10 points): Evaluate the integral of the following tabular data with the trapezoidal rule

| $x$ | -2 | 0 | 2 | 4 | 6 | 8 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 34 | 5 | -10 | 2 | 4 | 3 | 19 |

Problem 3 (20 points): Use order of $h^{8}$ Romberg integration to evaluate $\int_{0}^{\pi / 2}(5-3 \sin x) d x$ and compare $\varepsilon_{t}$ on the basis of the analytical solution.

Problem 4 (20 points): Obtain an estimate of the integral $\int_{0}^{\pi / 2}(5-3 \sin x) d x$ using threepoint Gauss-Legendre formula. Compute $\varepsilon_{t}$ on the basis of the analytical solution.

Problem 5 ( 40 points): Using each of the following methods, solve the following problem over the interval from $x=0$ to 1.5 using a step size of 0.5 where $y(0)=1$.

$$
\frac{d y}{d x}=(2+4 x) \sqrt[3]{y}
$$

(a) ( $\mathbf{1 0}$ points) Euler's method.
(b) ( $\mathbf{1 0}$ points) Midpoint method.
(c) (10 points) Heun's method without the corrector.
(d) ( $\mathbf{1 0}$ points) Fourth-order RK method.

Problem 6 ( $\mathbf{3 0}$ points): Solve the following problem with the third-order RK method:

$$
\frac{d^{2} y}{d x^{2}}+0.5 \frac{d y}{d x}+7 y=0
$$

where $y(0)=4$ and $y^{\prime}(0)=0$. Solve from $x=0$ to 2 with $h=0.5$.

