King Fahd University of Petroleum and Minerals College of Computer Sciences and Engineering

CISE 301 – Numerical Methods (T152)

Homework # 06 (due date & time: Sunday 03/04/2016 during class period)

*** Show all your work. No credit will be given if work is not shown! ***

Problem 1 (40 points): Use centered difference approximations of $O(h^4)$ to estimate the <u>first</u> and <u>second</u> derivatives for each of the following functions at the specified location and for the specified step size:

| (a) (20 points) $y = x^3 + 3x - 13$ | at $x = 0, h = 0.25$ |
|---|----------------------------------|
| (b) (20 points) $y = x^2 \sin x$ | at $x = 0.2$ (radian), $h = 0.1$ |

Problem 2 (30 points):

- (a) (25 points) Compute forward and backward difference approximations of O(h) and $O(h^2)$, and central difference approximations of $O(h^2)$ and $O(h^4)$ for the first derivative of $y = e^x$ at x = 0.1 for h = 0.2.
- (b) (5 points) Estimate the true percent relative error ε_t for each approximation.

Problem 3 (30 points): Consider the following data set:

| X | 0 | 2 | 3 | 3.5 | 4 | 4.5 | 5 | 6 | 7 |
|------|-----|---|------|-----|-----|-----|------|---|-----|
| f(x) | 0.5 | 1 | 0.75 | 4 | 3.5 | 1.9 | 1.35 | 1 | 1.7 |

- (a) (25 points) Using a tabular form, determine the best estimate of the first order derivative of f(x) at x = 4 using the Richardson extrapolation method with the highest accuracy possible. (Hint: Determine the appropriate *h* from the given data set)
- (b) (5 points) Determine the order, in terms of *h*, of the truncation error of part (a).