COE 202- Digital Logic

Number Systems II

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Objectives

- Base Conversion
  - Decimal to other bases
  - Binary to Octal and Hexadecimal
  - Any base to any base

Converting Decimal Integers to Binary

- Divide the decimal number by ‘2’
- Repeat division until a quotient of ‘0’ is produced
- The sequence of remainders in reverse order constitute the binary conversion
- Example: $(41)_{10} = (101001)_2$

LSB

$\frac{41}{2} = 20$	Remainder = **1**
$\frac{20}{2} = 10$	Remainder = **0**
$\frac{10}{2} = 5$	Remainder = **0**
$\frac{5}{2} = 2$	Remainder = **1**
$\frac{2}{2} = 1$	Remainder = **0**
$\frac{1}{2} = 0$	Remainder = **1**

MSB

Stopping Condition

Verify: $1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (41)_{10}$

Another Procedure for Converting from Decimal to Binary

- Start with a binary representation of all ‘0’s
- Determine the highest possible power of two that is less or equal to the number
- Put a ‘1’ in the bit position corresponding to the highest power of two found above
- Subtract the highest power of two found above from the number
- Repeat the process for the remaining number

Another Procedure for Converting from Decimal to Binary

- Example: Converting $(76)_{10}$ to Binary
  - The highest power of 2 less or equal to 76 is 64, hence the seventh (MSB) bit is 1
  - Subtracting 64 from 76 we get 12.
  - The highest power of 2 less or equal to 12 is 8, hence the fourth bit position is 1
  - We subtract 8 from 12 and get 4.
  - The highest power of 2 less or equal to 4 is 4, hence the third bit position is 1
  - Subtracting 4 from 4 yield a zero, hence all the left bits are set to 0 to yield the final answer

Decimal to binary conversion chart

Decimal	binary equivalent (3-bits)	binary equivalent (4-bits)
0	000	0000
1	001	0001
2	010	0010
3	011	0011
4	100	0100
5	101	0101
6	110	0110
7	111	0111
8	Not convertible (need more bits)	1000
9	Not convertible (need more bits)	1001
10	Not convertible (need more bits)	1010
11	Not convertible (need more bits)	1011
12	Not convertible (need more bits)	1100
13	Not convertible (need more bits)	1101
14	Not convertible (need more bits)	1110
15	Not convertible (need more bits)	1111
16	Not convertible (need more bits)	Not convertible (need more bits)

Converting Decimal Integer to Octal

- Divide the decimal number by ‘8’
- Repeat division until a quotient of ‘0’ is received
- The sequence of remainders in reverse order constitute the octal conversion
- Example: $(153)_{10} = (231)_8$

$\frac{153}{8} = 19$	Remainder = **1**
$\frac{19}{8} = 2$	Remainder = **3**
$\frac{2}{8} = 0$	Remainder = **2**

LSB

MSB

Stopping Condition

Verify: $2 \times 8^2 + 3 \times 8^1 + 1 \times 8^0 = (153)_{10}$

Converting Decimal Integer to Hexadecimal

- Divide the decimal number by ‘16’
- Repeat division until a quotient of ‘0’ is received
- The sequence of remainders in reverse order constitute the hexadecimal conversion
- Example: $(422)_{10} = (1A6)_{16}$

Converting Decimal Fraction to Binary

- Multiply the decimal number by '2'
- Repeat multiplication until a fraction value of '0.0' is reached or until the desired level of accuracy is reached
- The sequence of integers before the decimal point constitute the binary number
- Example: $(0.6875)_{10} = (0.1011)_2$

MSB

$0.6875 \times 2 = \mathbf{1}.3750$

$0.3750 \times 2 = \mathbf{0}.7500$

$0.7500 \times 2 = \mathbf{1}.5000$

$0.5000 \times 2 = \mathbf{1}.0000$

LSB

0.0000

Stopping Condition

Verify: $1\text{x}2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} = (0.6875)_{10}$

Converting Decimal Fraction to Octal

- Multiply the decimal number by ‘8’
- Repeat multiplication until a fraction value of ‘0.0’ is reached or until the desired level of accuracy is reached
- The sequence of integers before the decimal point constitute the octal number
- Example: $(0.513)_{10} = (0.4065\ldots)_8$

MSB

0.513 x 8 = **4**.104

0.104 x 8 = **0**.832

0.832 x 8 = **6**.656

0.656 x 8 = **5**.248

LSB

. . . .

Verify: $4 \times 8^{-1} + 0 \times 8^{-2} + 6 \times 8^{-3} + 5 \times 8^{-4} = (0.513)_{10}$

Converting both Integer & Fraction

- Question. How to convert a decimal number that has both integral and fractional parts?
- Answer. Convert each part separately, combine the two results with a point in between.
- Example: Consider the "decimal -> octal" examples in previous slides
  - $(153.513)_{10} = (231.407)_8$

Example

- Convert $(211.6250)_{10}$ to binary?
  - Steps:
    - Split the number into integer and fraction
    - Perform the conversions for the integer and fraction part separately
    - Rejoin the results after the individual conversions

Example

$\frac{211}{2} = 105$	Remainder = 1
$\frac{105}{2} = 52$	Remainder = 1
$\frac{52}{2} = 26$	Remainder = 0
$\frac{26}{2} = 13$	Remainder = 1
$\frac{13}{2} = 6$	Remainder = 1
$\frac{6}{2} = 3$	Remainder = 0
$\frac{3}{2} = 1$	Remainder = 1
$\frac{1}{2} = 0$	Remainder = 1

Integer part

fraction part

MSB

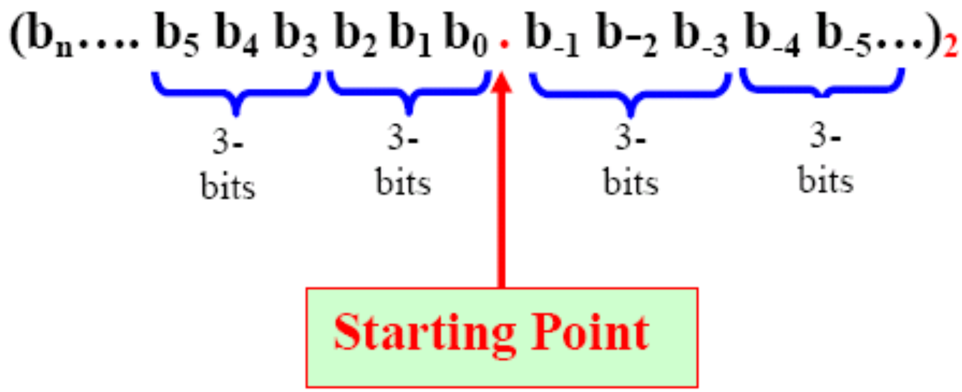
$$0.6250 \times 2 = 1.25$$
$$0.2500 \times 2 = 0.50$$
$$0.5000 \times 2 = 1.00$$

LSB

Combining the results gives us:

$$\mathbf{(211.6250)_{10} = (11011011.101)_2}$$

Converting Binary to Octal



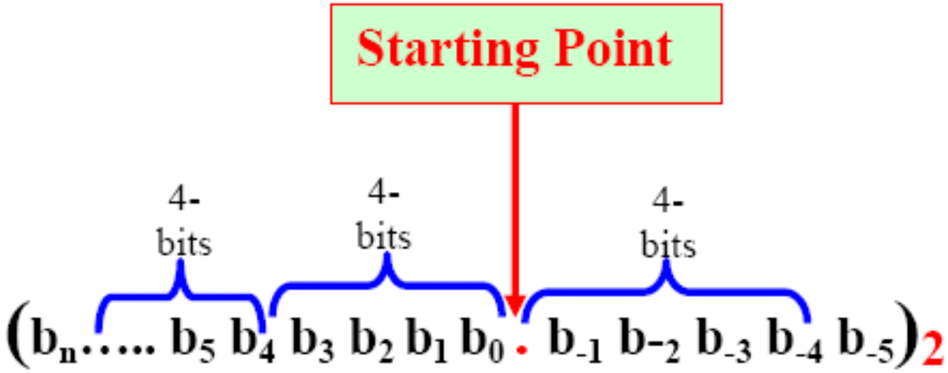
Group of 3 Binary Bits $b_{i+2}\ b_{i+1}\ b_i$	Octal Equivalent
0 0 0	0
0 0 1	1
0 1 0	2
0 1 1	3
1 0 0	4
1 0 1	5
1 1 0	6
1 1 1	7

- Example: Convert $(1110010101.1011011)_2$ into Octal.

001__110__010__101_._101__101__100 $= (1625.554)_8$

1 6 2 5 5 5 4

Binary to Hexadecimal Conversion



Group of 4 Binary Bits $b_{i+3}\ b_{i+2}\ b_{i+1}\ b_i$	Hexadecimal Equivalent
0 0 0 0	0
0 0 0 1	1
0 0 1 0	2
0 0 1 1	3
0 1 0 0	4
0 1 0 1	5
0 1 1 0	6
0 1 1 1	7
1 0 0 0	8
1 0 0 1	9
1 0 1 0	A
1 0 1 1	B
1 1 0 0	C
1 1 0 1	D
1 1 1 0	E
1 1 1 1	F

- Example: Convert $(1110010101.1011011)_2$ into hex.

0011__1001__0101_._1011__0110 $= (395.B6)_{16}$

3 9 5 B 6

Converting between other bases

- Question. How to convert between bases other than decimal; e.g. from base-4 to base-6?
- Answer. Two steps:
  - convert source base to decimal
  - convert decimal to destination base

Converting Hexadecimal to Octal (special case)

- In this case, we can use binary as an intermediate step instead of decimal
- Example:
  - $(3A)_{16} = (0011\text{-}1010)_2 = (000\text{-}111\text{-}010)_2 = (072)_8$

0 added

re-group by 3

Converting Octal to Hexadecimal (special case)

- In this case, we can use binary as an intermediate step instead of decimal

- Example:
  - $(72)_8 = (111\text{-}010)_2 = (0011\text{-}1010)_2 = (3A)_{16}$

0 added

re-group by 4

Conclusions

- To convert from decimal to base-r, divide by r for the integral part, multiply by r for the fractional part, then combine
- To convert from binary to octal (hexadecimal)
- group bits into 3 (4)
- To convert between bases other than decimal, first convert source base to decimal, then convert decimal to the destination base.