KFUPM – CCSE - COMPUTER ENGINEERING DEPARTMENT

CSE 642 – Computer Systems Performance (Take home quiz 1)

Student Name:
Student Number:

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1) (**10 points**) Let X be a non-negative integer valued random variable, show that

$$E[X] = \sum_{k=0}^{\infty} P(X > k):$$

2) (**5 points**) Prove the identity given in (1) for a Geometric random variable whose PMF is given by $\text{Prob}[X = k] = (1-p)^{k-1} p$ for k = 1, 2, … and 0 <p <1.

3) (**5 points**) Find the probability generating function (PGF) for the geometric distribution specified in (2).

4) Consider a normal random variable X with mean μ and standard deviation σ.

a) (**5 points**) Specify and plot the PDF for X, $f_X(x)$, for μ equal to 0 and σ equal to 1, 2, and 3. (all three curves on one plot with proper legend).

b) (**5 points**) Define the random variable Y as $y = \exp(x)$, then Y is said to follow the lognormal distribution. Specify and plot the PDF for Y, $f_Y(y)$, for the three given sets of parameters listed in (4.a).

c) (**bonus 10 points**) The characteristic function (CF) for the random variable Y, $\Phi_Y(\omega)$, can be computed using $\Phi_Y(\omega) = \int_{y=0}^{\infty} e^{j\omega y} f_Y(y)\,dy$ where $f_Y(y)$ is the PDF evaluated in part (4.b). The CF can also be evaluated using $\Phi_Y(\omega) = \int_{-\infty}^{\infty} e^{j\omega e^x} f_X(x)\,dx$ where $f_X(x)$ is the PDF for the corresponding normal random variable (i.e. specified in part 4.1). It is required to evaluate and plot $\Phi_Y(\omega)$ for $\omega \in [0,10^4]$ and for the three given sets of parameters listed in (4.a). Use the log scale for both the CF axis and the frequency axis ω. Choose evenly spaced points on the x-axis using the Matlab command "`logspace`". Provide one plot for $\left|\text{real}(\Phi_Y(\omega))\right|$ and separate plot for $\left|\text{imag}(\Phi_Y(\omega))\right|$. *Comment* on the integrand $e^{j\omega e^x} f_X(x)$ and the relative ease or difficulty of computing $\Phi_Y(\omega)$.

Hint: Matlab is recommended for the plotting and evaluation of functions.