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- Fixed length frame, T_F includes all slots, guard time, etc.
- A source may transmit UP TO b >= 1 slots during a frame
- Gated service as opposed to exhaustive. Packets arriving during the frame must wait until the next frame
- The equation for the embedded Markov chain for the number of packets in the buffer is given by

 $N_{i+1} = max(0, N_i-b) + A_{i+1}$

where N_i is the number of packets in the system at the beginning of the ith frame, A_i is the number of packets arriving during the ith frame

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Synchronous Time-Division Multiplexing (STDM) - Analysis - cont'd The objective is to compute the PGF for the number packets in the system $P_{i+1}(z) = E(z^{N_{i+1}}) = E(z^{\max(0,N_i-b)+A_{i+1}}) = E(z^{\max(0,N_i-b)})E(z^{A_{i+1}})$ After simple manipulation $P_{i+1}(z) = \left[\sum_{j=0}^{b-1} P_j^i + \sum_{j=b}^{\infty} P_j^i z^{j-b}\right] E[z^{A_{i+1}}]$ Assume the steady state solution exists and that Lim Pji = Pj, then
$$\begin{split} P(z) = & \left[\sum_{j=0}^{b-1} P_j + z^{-b} \left(\sum_{j=b}^{\infty} P_j z^j - \sum_{j=0}^{b-1} P_j z^j\right)\right] A(z, T_F) \\ & = A(z, T_F) \sum_{j=0}^{b-1} P_j \left(1 - z^{-b+j}\right) + z^{-b} A(z, T_F) P(z) \end{split}$$
where A(z, TF) is the PGF for the number of arriving packets during a frame Therefore, the required PGF is given by $P(z) = \frac{A(z,T_F) \sum_{j=0}^{b-1} P_j(z^b - z^j)}{z^b - A(z,T_F)}$ Main result There are b unknowns: P0, P1, ..., Pb-1 Note that if b = 1, the above PGF reduces the one on slide 26!! 1/5/2010 Dr. Ashraf S. Hasan Mahmoud 32



- How to find the unknowns P0, P1, ..., Pb-1?
- If it was only for P0 (i.e. b = 1), then the condition that P(1) (i.e. PGF evaluated at z = 1) must be 1 is sufficient to find the unknown P0.
- For the case of b > 1, we need extra b-1 conditions or equations to find P1, P2, ..., Pb-1.
- These equations can be obtained by applying Rouche's theorem.
 - The provided solution restricts the arrival process to Poisson
 - But the concept is applicable to any arrival process.

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Synchronous Time-Division Multiplexing (STDM) - Rouche's Theorem Consider the properties of the PGF P(z), z^{b} , and A(z, T_F). P(z) is analytic within the unit disk $|z| \le 1$ $\left|P(z)\right| = \left|\sum_{i=0}^{\infty} z^{j} P_{j}\right| \le \sum_{i=0}^{\infty} \left|z^{j}\right| P_{j} \le \sum_{i=0}^{\infty} P_{j} = 1$ The arrival process is assumed to Poisson; then $A(z,T_{\rm F}) = \exp(-\lambda T_{\rm F}(1-M(z)))$ For stability, we require that average packet arrivals not exceed b. $b > \lambda T_F M'(1) = \lambda T_F \overline{M}$ Therefore, where \overline{M} is the mean number of packets in a message. Now consider the zeros, z_i 's where $|z_i| \le 1$ for the denominator of P(z) $z_i^b = A(z_i, T_F) = e^{-\lambda T_F(1-M(z_i))}$ The textbook shows that for all these z_i's the root MUST be simple 1/5/2010 Dr. Ashraf S. Hasan Mahmoud 34









