KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
COLLEGE OF COMPUTER SCIENCES & ENGINEERING
COMPUTER ENGINEERING DEPARTMENT
CSE 642 – Computer Systems Performance
Assignment 1 – Due date: Nov 2nd, 2009

Problem 1 (10 points):

Calculate the expected value for a bounded Pareto distribution in terms of α (the shape parameter), β (the scale parameter), and $S_{\max}$, the maximum packet size in bytes. For this problem assume that the PDF is given by $f_X(x) = c\frac{\alpha\beta^{\alpha}}{x^{\alpha+1}}$ for $\beta \le x \le S_{\max}$. The constant c is chosen such that $f_X(x)$ is a *valid* PDF.

Problem 2 (20 points):

Let Z = X + Y where X and Y are two independent continuous exponential random variables with parameters α and β, respectively. Find the PDF for the Z through:

a) Calculating the join PDF for the variables X and Y.
b) Inverting the product of two individual characteristic functions.

Problem 3 (40 points):

Consider the random variable Z_i that is the sum of i (i = 1, 2, 3, …) independent and identically distributed lognormal variables. Let the parameters for the individual lognormal random variable, X, be μ and σ that are equal to 0 and 2.76, respectively. The closed form PDF for Z is not known.

a) The CDF for X can be easily evaluated using the Q function. It is required to plot the CDF of X on two different graphs. The first graph is a typical CDF graph where the CDF function is plotted using the Matlab `semilogy` function. Typically also, for the lognormal RV, we use `log(X)` as the x-axis, since X can take very small and very large values. The second graph is again a plot for the same CDF function but on a "normal probability paper". The code for doing such a plot will be explained in class. Limit the y-axis for the first graph to the range of 10^{-6} to 1, and to the range of 10^{-6} to $1\text{-}10^{-6}$ for the second graph.

   You are to submit the two separate graphs.

b) Evaluate the CDF of Z_i empirically using Matlab for i = 1, 6, 10, and 20. Use 10^6 or more samples for each of the individual lognormal random variables (i.e. the X's). Plot the CDF for Z_i's on one normal probability plot. State your observations in regard to the shape of the CDF curve as i increases. Compare your results with the following reference:

   N. C. Beaulieu and F. Rajwani, "Highly accurate simple closed-form approximations to lognormal sum distributions and densities," *IEEE Commun. Letters*, vol. 8, no. 12, pp. 709-711, Dec 2004.

c) For Z_6, use the Markov inequality and Chebyshev bound to approximate the Probability the complementary CDF (i.e. 1 minus the CDF). Plot the empirical CCDF and the two evaluated bounds on normal probability paper.

d) Evaluate the characteristic function (CF) for the RV Z_6 and utilize it to compute the Chernoff bound for the CCDF. Provide a plot for the CCDF and the bound on a normal probability paper.

For all parts of this problem, you are also required to submit the matlab script used to do the calculations and the plots.

Problem 4 (10 points):

Prove that for a Poisson arrival process of mean λt, the interarrival time is an exponential random variable of mean $1/\lambda$.

Problem 5 (20 points):

Consider a discrete-time traffic batched arrival process where the probability of a batch arrival in a given time slot is equal to $0< p <1$. An arriving batch contains a random number of packets that is geometrically distributed with mean equal to 3 packets.

a) Calculate the mean and the variance of the number of arriving *batches* in a given time slot.

b) Calculate the mean and variance of the number of arriving *packets* in a given time slot.

c) Compute the probability generating function for the number of arriving packets in a given time slot.