New Optimization Criteria for Message-Switching Networks

BERND MEISTER, H. R. MÜLLER, AND HARRY R. RUDIN, JR., MEMBER, IEEE

Abstract—**This paper presents a new class of criteria for the optimum capacity assignment in store-and-forward communication networks under a total fixed-cost constraint. Compared with conventional average-delay optimization these criteria are more sensitive to the needs of the individual user. Closed-form results are attained.**

Introduction

THIS PAPER DEALS with the optimization of a store-and-forward (or message-switching) communication network. The location of the switching nodes and complete traffic description—density, origin, destination, and routing of traffic—are known. The usual distributions are assumed—exponential for both interarrival and message-length distributions. In addition, the message-length independence assumption of Kleinrock [1] is made to permit mathematical tractability.[1] The analysis to follow holds only for the steady state.

The usual design procedure as first given by Kleinrock [1] is to minimize the average message delay through the network given a prescribed cost. This is a good approach but has the disadvantage that some individual users may suffer very long delays. Here an approach is suggested which provides better treatment for the individual user at only a moderate expense to the global-system performance as measured by average message delay.

The former approach, that of Kleinrock, is very fine for the design of a telegraph network for which it was initially intended, where the main concern may well be that of overall system performance. In the case of a network for data transmission, wherein the data may often be used in an interactive fashion, it would seem appropriate to carefully examine the effect on the individual user of any overall optimization scheme and further to see what can be done for the individual user and at what expense. New criteria are introduced here which are better tailored to the data-network problem and are mathematically tractable as well.

Specifically, it will be shown that by minimizing the appropriately weighted sum of powers of the average message delay in the various links, the variation in delay from link to link can be substantially reduced. Furthermore, it will be shown that the variance of the message delay (the deviation from the mean seen on a link) can also be minimized. In both cases the prescribed total cost constraint is satisfied.

Notation and Definitions

A number of definitions are necessary before proceeding. The letter i denotes a particular line or link; $1 \leq i \leq N$.

Given:

- λ_i Average of the Poisson traffic entering the ith line (packets per second).
- $1/\mu_i$ Average packet length for line i (bits per packet).
- $\bar{n}$ Number of links used by a message averaged over all messages.
- d_i Cost per unit capacity of line i (dollars per bit per second per month).
- ρ_i Average utilization of line i.
- γ Average of total created traffic (packets per second). This is the total flow of messages into the network.
- D Total cost (dollars per month).

To be calculated:

- C_i Capacity of line i (bits per second).
- T_i Average delay in line i (seconds).
- T Average delay (seconds).

Having made these definitions, several relations can be stated which will be used later on.

The total average traffic γ entering the network, multiplied by the average number of links traversed by a typical message gives the total link traffic:

$$\bar{n}\gamma = \sum \lambda_i.$$

The proportion of the link capacity which is used in the mean will be called line utilization and is given by

$$\rho_i = \frac{\lambda_i}{\mu_i C_i}.$$

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The authors are with the IBM Research Laboratory, Rüschlikon, Switzerland.

[1] The message-length independence assumption means that for each message entering a node, a new length is selected from an exponentially distributed set of lengths. Simulations [1] have indicated that this assumption does not impact the utility of the numerical results.

The average delay T_i in the ith line including service time per message T_s and waiting time is [2]

$$T_i = \frac{T_s}{1 - \rho_i}$$

assuming that the arrivals are Poisson and message transmission times are exponential.

Since service time

$$T_s = \frac{1}{\mu_i C_i}$$

T_i can be rewritten as

$$T_i = \frac{1}{\mu_i C_i - \lambda_i}. \tag{1}$$

These delays T_i do not include propagation time along the lines. These propagation delays will be neglected throughout the paper because they do not enter the optimization procedure.

Given the average delay per link, a total average delay can be calculated by weighting each link delay by the proportion of total traffic carried by it:

$$T = \sum \frac{\lambda_i}{\gamma} T_i. \tag{2}$$

A further definition can now be made, that of the minimum feasible network cost. This figure is obtained by assigning to each line the very minimum capacity, i.e., $\rho_i = 1$, for all i. This assignment would mean that the delays for the system approach infinity on every link. This minimum network cost is given by

$$D^* = \sum d_i \frac{\lambda_i}{\mu_i}.$$

Since a total of D dollars per month are available for the network and since D must be greater than D^*, one can also define the "freely-assignable" portion of the total cost

$$D_a = D - D^*.$$

Proceeding further, it is possible to determine the utilization of money spent for the network as simply the ratio of the minimum cost to actual cost. This "capital utilization" is D^*/D.

The optimization procedure begins with specification of the total cost or, equivalently, the capital utilization. It is possible to define an average line utilization in the same fashion, i.e., that fraction of available capacity actually needed. Thus the average utilization ρ is defined as

$$\rho = \frac{\sum \lambda_i/\mu_i}{\sum C_i}. \tag{3}$$

From the assumptions made, it follows that the delay of line i has a negative exponential distribution, so that the variance of this delay is equal to the mean value squared, i.e., $(T_i)^2$. Since line delays are considered independent random variables, the expression

$$\sum \frac{\lambda_i}{\bar{n}\gamma} (\bar{n}T_i)^2$$

can be considered a weighted variance of the overall delay. Reference will be made here to the weighted standard deviation of the overall delay

$$\sigma(T) = \left[\sum \frac{\lambda_i \bar{n}}{\gamma} (T_i)^2\right]^{1/2}. \tag{4}$$

It is also important to consider the spread of the average delay over the various links given by

$$S(T) = \left[\sum \frac{\lambda_i}{\bar{n}\gamma} (\bar{n}T_i - T)^2\right]^{1/2}. \tag{5}$$

The spread then is a measure of the uniformity of service over the various links. A large spread in a network with reasonable global performance indicates that delay times on some links are substantially shorter than they need be, and what is more detrimental, that on other links the delays may be intolerably long. It is this spread in particular which this paper seeks to alleviate, relative to the conventional design.

Criteria for Optimization

As stated earlier, the usual criterion is minimization of the average message delay. One might also consider minimizing the variance of the overall delay. What is proposed here is a generalization of these ideas—minimization of the mean kth power. Thus the class of criteria proposed is given by allowing k to take on integer values in the relation

$$T^{(k)} = \left[\sum \frac{\lambda_i}{\gamma} (T_i)^k\right]^{1/k}. \tag{6}$$

As is usual, the capacities of the various links C_i are then chosen to minimize the objective function $T^{(k)}$ under the constraint of fixed total cost

$$D = \sum d_i C_i. \tag{7}$$

It is clear that, for $k = 1$, the usual criterion of Kleinrock [1] is attained. It may also be seen that, for $k = 2$, the weighted variance of message delay is minimized because the mean and standard deviation of a random variable are equal if that random variable is described by an exponential distribution. If k is allowed to become very large, approaching infinity in the limit, optimization under this objective function is equivalent to a Chebyshev or minimax criterion and all the individual average link delays become equal in the solution $T^{(\infty)} = \max_i T_i$.

The authors feel this class of criteria is advantageous, particularly for large values of k. For such values, the raising of an above-average link delay to a high power results in allocation of extra capacity to that link. In the normal case, with $k = 1$, very light traffic on a link makes any delay on that link, even a very large one, almost invisible to the optimization procedure. This will become evident in an example.

Analytic Results[2]

Using the method of Lagrange multipliers, a simple solution for the optimization problem is obtained. First, the Lagrangian multiplier itself can be shown to be

$$L^{(k)} = \left\{ \frac{\sum_i (k\lambda_i d_i^k/\gamma\mu_i^k)^{1/(k+1)}}{D_a} \right\}^{(k+1)}. \quad (8)$$

The superscript k indicates the dependence on the parameter of the objective function. The individual link capacities $C_i^{(k)}$ can be expressed in terms of the Lagrangian multiplier:

$$C_i^{(k)} = \frac{\lambda_i}{\mu_i} + \left(\frac{k\lambda_i}{\gamma d_i \mu_i^k L^{(k)}}\right)^{1/(k+1)}. \quad (9)$$

For the case of $k = 1$, as stated earlier, one gets the square-root-channel capacity assignment first described by Kleinrock [1]. For the case of $k = 2$, the variance is minimized, as discussed previously. For k approaching zero, included mainly for completeness, one gets the following simple relationships:

$$C_i^{(0)} = \frac{\lambda_i}{\mu_i} + \frac{\lambda_i D_a}{\gamma d_i \bar{n}}$$

and for the delays,

$$T_i^{(0)} = \frac{\gamma d_i \bar{n}}{\mu_i \lambda_i D_a}.$$

The case $k = 0$ corresponds to the proportional capacity assignment of Kleinrock [1] when all cost factors d_i and packet lengths $1/\mu_i$ are the same.

A more interesting case is the other limiting case, that of k approaching infinity. Here one obtains, for the capacities,

$$C_i^{(\infty)} = \frac{\lambda_i}{\mu_i} + \frac{1}{\mu_i} \frac{D_a}{\sum_j (d_j/\mu_j)} \quad (10)$$

and for the delays,

$$T_i^{(\infty)} = \frac{1}{D_a} \sum_j \frac{d_j}{\mu_j}. \quad (11)$$

Note that the right-hand side of (11) does not depend on i; in this case all delays are equal and the average delay $T^{(\infty)}$ becomes

$$T^{(\infty)} = \frac{\bar{n}}{D_a} \sum_j \frac{d_j}{\mu_j}. \quad (12)$$

Numerical Results

In the following several of the criteria previously mentioned will be compared using the example of a telegraph network appearing in Kleinrock (see especially [1, pp. 22–23]). In this example the line cost factors d_i are all considered equal to unity and all package lengths $1/\mu_i$ are also equal. The average traffic on the seven links is shown in Table I. This traffic is the same for all examples in this paper. Table I also shows the capacity assignments for various values of k. The lines with lightest and heaviest traffic are noted. In all cases the average line utilization is 25 percent, identical to the capital utilization.

As k is increased the line capacities become more uniform. Finally, for $k \to \infty$, each line has been assigned an identical amount of additional capacity over the minimum required because all μ_i are identical.

Figs. 1–6 show the behavior of the various performance criteria for several values of k. For a particular value of k, the optimum $C_i^{(k)}$ are calculated; the performance criteria are then evaluated. This variation is shown for three different average utilizations. Looking first at any of the sets of curves in Figs. 1, 3, or 5, it may be seen that as k approaches infinity, the average delay for the entire network increases, but only by 20 percent in the cases shown here. The average delay for the line with the highest traffic load increases substantially, but in a region of delay sufficiently small so that the user is unlikely to detect a degradation of performance. On the other hand, the line with the smallest traffic load experiences a markedly decreased delay and this occurs in a region where the user would be very much aware of what goes on.

Note that, for $k = \infty$, the average delay is equal to the individual line delays multiplied by $\bar{n}$, where $\bar{n}$ is the average number of links passed by a message. ($\bar{n} = 1.31$ in the example discussed here.) Optimization for $k = 0$ produces large values which do not fit conveniently on the curves but are included in Table II for completeness.

A point which should be reiterated is that the standard deviation and the mean of the delay for a single line are equal for the case of the Poisson statistics assumed here. This means that a large average link delay implies a wide variation in the actual delay times for that link. The very long delays resulting from this large variance would be intolerable to the user.

It can also be seen from Figs. 2, 4, and 6 that, for very large k, the spread of delay times approaches zero. This follows also from Figs. 1, 3, and 5 as the curves for links with minimum and maximum delay approach each other. The decrease in spread is indicative of the greater uniformity of service attained.

From Fig. 7 it may be seen that the global weighted standard deviation does not depend heavily on the parameter k of the objective function. This is a pleasing result because it shows that the price paid for using higher values of k in the objective function is small indeed: only slight increases in average delay and standard deviation result. In contrast, one obtains sharply reduced delays on the links, which would have had abnormally long delays in the conventional design procedure.

[2] The derivation of these results is briefly outlined in Appendixes I and II.

TABLE I

LINE CAPACITIES FOR SEVERAL k AND AVERAGE UTILIZATION $\rho = 0.25$

		Line Number						
		1	2	3	4[a]	5	6[b]	7
Load[c]		3.15	3.64	3.55	0.13	0.82	9.95	3.88
Capacity[c]	$k = 1$	14.34	15.66	15.42	2.41	6.53	29.83	16.28
	$k = 2$	14.55	15.59	15.40	4.08	8.09	26.67	16.08
	$k = 4$	14.49	15.30	15.15	6.13	9.48	24.22	15.69
	$k = 8$	14.31	14.98	14.85	7.97	10.43	22.63	15.29
	$k = 16$	14.15	14.73	14.62	9.25	10.98	21.72	15.01
	$k = 32$	14.04	14.48	14.48	10.02	11.28	21.23	14.83
	$k = 64$	13.98	14.49	14.40	10.44	11.43	20.97	14.74
	$k = \infty$	13.92	14.40	14.31	10.89	11.58	20.71	14.64

[a] Line with lightest traffic.
[b] Line with heaviest traffic.
[c] In packets per second.

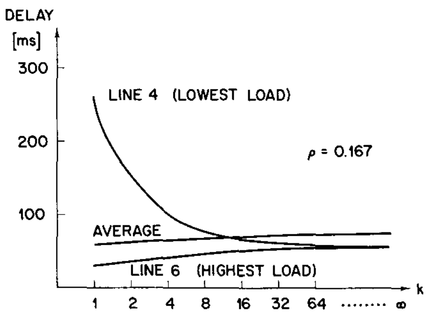

Fig. 1. Delay for 16.7 percent average utilization.

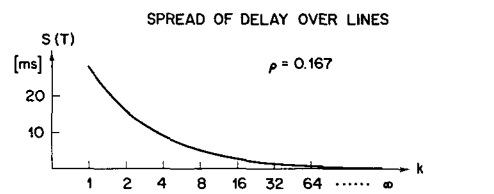

Fig. 2. Spread for 16.7 percent average utilization.

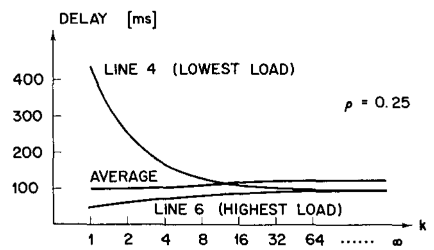

Fig. 3. Delay for 25 percent average utilization.

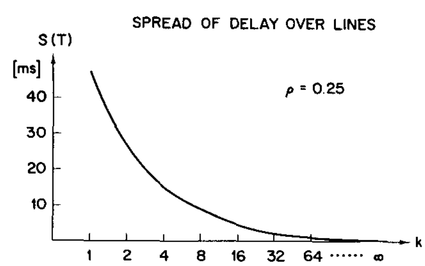

Fig. 4. Spread for 25 percent average utilization.

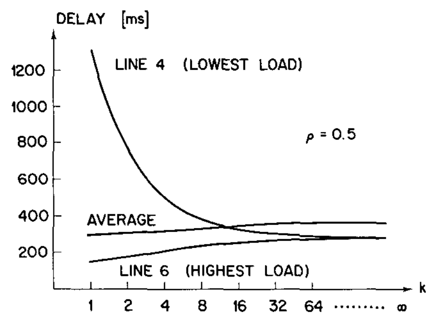

Fig. 5. Delay for 50 percent average utilization.

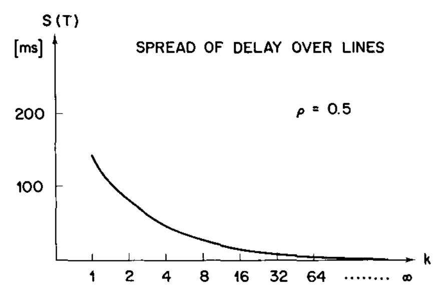

Fig. 6. Spread for 50 percent average utilization.

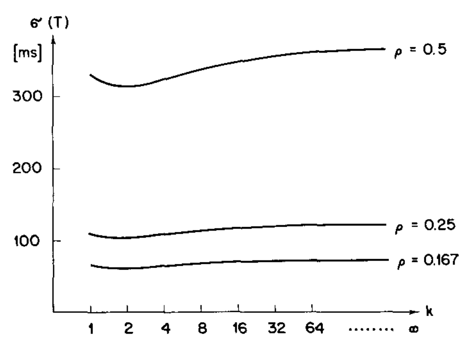

Fig. 7. Weighted standard deviation for several average utilizations.

TABLE II
$k = 0$[a]

Utilization ρ	Average Delay (ms)	Variance (ms)	Spread Over Lines (ms)
0.167	73	166	149
0.25	122	277	249
0.5	365	831	746

[a] In this example the case $k = 0$ is equivalent to proportional capacity assignment.

Conclusion

In summary, a new class of criteria has been proposed for optimization of store-and-forward data communication networks. This class of functions results in capacity assignments which are better tailored to data communication network design because more attention is paid to the needs of the individual user. This results in 1) a smaller spread of delay between links and, consequently, 2) reduced statistical variation of delay on the more lightly loaded links. Furthermore, these advantages can be had with only a moderate increase in average delay.

Appendix I
$k \to \infty$

It will be shown that $T^{(k)}$ converges to $\max_i T_i$ as k tends towards infinity. Let m be an index for which $T_m \geq T_i$, $i = 1,\cdots,N$. This index m might not be unique but can be chosen to be the same for all k. Next, the following simple inequalities are considered:

$$\left(\frac{\lambda_m}{\gamma}\right)^{1/k} T_m \leq \left(\sum \frac{\lambda_i}{\gamma} (T_i)^k\right)^{1/k}$$

$$= T_m \left(\sum \frac{\lambda_i}{\gamma} \left(\frac{T_i}{T_m}\right)^k\right)^{1/k} \leq T_m(\bar{n})^{1/k}.$$

Since the bounds for the objective function tend to T_m as $k \to \infty$, the desired result has been established.

Appendix II
Optimization of $C_i^{(k)}$

The minimization of $T^{(k)}$ is obviously equivalent to minimizing $(T^{(k)})^k$ and the problem becomes

$$\text{minimize } \sum \frac{\lambda_i}{\gamma} (T_i)^k, \qquad \text{subject to } \sum d_i C_i^{(k)} = D.$$

It can be readily seen that the objective function is a sum of convex functions with positive coefficients and, therefore, is itself convex. This guarantees existence of a unique minimum. With a Lagrange multiplier [3] $L^{(k)}$ to be determined later, the following system of equations may be set up:

$$\frac{\partial}{\partial C_i(k)} \left(\sum \frac{\lambda_i}{\gamma} (T_i)^k + L^{(k)} \cdot \sum d_i C_i^{(k)}\right) = 0$$

where $i = 1,\cdots,N$, or

$$\frac{\mu_i \lambda_i}{\gamma} \frac{k}{(\mu_i C_i^{(k)} - \lambda_i)^{k+1}} = L^{(k)} d_i.$$

Solving for $C_i^{(k)}$ results in (9). Multiplication of the latter equations by d_i and summation over all i gives the one additional equation which is required to obtain $L^{(k)}$ since $\sum d_i C_i^{(k)}$ is known to be equal to D.

References

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[2] R. Syski, *Introduction to Congestion Theory in Telephone Systems.* London: Oliver & Boyd, 1960, p. 299.
[3] R. Courant, *Differential and Integral Calculus,* vol. II. New York: Interscience, 1966, pp. 188–199.

Bernd Meister was born in Chemnitz, Germany, on January 28, 1936. He received the M.S. degree in mathematics from Humboldt University, Berlin, Germany, in 1958 and the Ph.D. degree in mathematics from the University of Freiburg, Germany, in 1962.

From 1962 to 1964 he was Scientific Assistant at the Institute for Applied Mathematics, University of Freiburg. Since 1964 he has been with the IBM Research Laboratory, Rüschlikon, Switzerland, and is now working in the areas of communication theory and queueing theory.

Dr. Meister is a member of the German Association for Applied Mathematics and Mechanics.

H. R. Müller was born in Istighofen, Switzerland, in July 1932. He received the M.S. degree in electrical engineering from the Swiss Federal Institute of Technology, Zurich, in 1957.

He joined the IBM Research Laboratory, Rüschlikon, Switzerland, in 1958, where he was doing research in the area of digital pneumatic elements. Beginning in 1961 his research concentrated on analysis of elements with no moving parts (fluidics). From 1964 to 1967 he worked on pneumo-mechanical I/O systems with miniature diaphragm devices in the Advanced Technology Group at the IBM SDD Development Laboratory in Endicott, N. Y. Since 1967 he has been engaged in communication research on integrated digital systems in the Systems Architecture Group at the IBM Research Laboratory, Rüschlikon.

Harry R. Rudin, Jr. (S'55–M'62), for a photograph and biography please see p. 187 of the April 1971 issue of this Transactions.