# Cell Site Power Characterization for Multi-rate Wireless CDMA Data Networks Using Lognormal Approximation

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***Abstract*—An analytical model is developed to solve the power-rate assignment problem for multi-rate CDMA systems and calculate the probability density function (PDF) for the downlink traffic power as a function of the bit rate assignment scheme, the target link quality figure, and the other network parameters. This paper proposes the use of the lognormal random variable as an approximation for an empirical parameter (denoted herein by $f_i$) that is needed to solve the energy per-bit to noise spectral power density equation for scheduled data bursts. The approximation depends on matching the mean and standard deviation of the lognormal variable to the mean and standard deviation of the empirical parameter $f_i$ where the matching depends only the radio frequency (RF) path loss model and the shadowing process standard deviation. The paper presents example calculations for the cumulative probability distribution (CDF) of the downlink traffic power and outage probabilities for a general CDMA system assuming specific scenarios of data bit rate assignments. The study also compares between analytical results and those obtained using conventional Monte-Carlo simulations. Comparisons indicate that the developed model provides an appropriate mechanism to approximate both the PDF or CDF, and the outage probability for the cell site total downlink traffic power.**

***Index Terms*—code division multiple access, radio resource management, call admission control, burst scheduling**

## I. Introduction

With the proliferation of wireless data services, there is an increasing need for the optimization and efficient design of radio resource management procedures to enhance the capacity and performance of wireless data networks. Specifically, recent developments in third Generation (3G) and other CDMA based systems highlight the need for efficient call admission control (CAC) and burst admission control as part of the radio resource management procedures. In this study we focus on the burst admission control and provide a model for the characterization of the downlink traffic power utilizing approximations using the lognormal random variable.

Being one of the widely accepted technologies, CDMA is deployed as the air interface for current 3G networks [1]. These networks should support heterogeneous services with variable quality of service (QoS) requirements. Unlike frequency division multiple access (FDMA) or time division multiple access (TDMA) schemes, CDMA does not have a hard capacity limit and the number of supported calls or bursts is a function of the radio frequency propagation and the subscriber location. Therefore, CDMA capacity studies that take into account the radio frequency propagation and the subscriber location are generally not possible using Markovian analysis. Typically, such studies resort to simulations to optimize the system data throughput. Examples of these studies can be found in [2][3][4][5] and the references therein. Some, but fewer, studies employing Markovian analysis do exist such as those in [6] and [7]. However, in there it is assumed that different calls with the same rate consume equal resources regardless of the subscriber location or the channel condition.

The study in [8] analyzes the performance of a single-cell single-rate CDMA system while approximating the interference by Gaussian random variable to enable the exploitation of Erlang capacity calculations for the system. The same approach is enhanced and utilized by [9] where the interference is approximated by an equivalent number of voice calls which was found to be closely approximated by a gamma random variable. The common two features of these two studies is first that only a single and relatively low service rate is assumed, and second the models do not account for the shadowing process as part of the radio frequency propagation model. This makes the number of potential calls large and thus the interference can be modeled by Gaussian or gamma distributions.

This paper proposes a model to characterize the downlink traffic power for multi-rate CDMA system. The model is based on power-rate assignment formulation that attempts to provide a specified target link quality figure. The developed model may serve as the analytical framework to enable CAC and RRM studies with the consideration of radio frequency propagation and the soft capacity issue for the downlink. The developed model approximates the propagation related parameter using a lognormal random variable and provides a characterization for the sum of lognormal variables to arrive at the complete specification of the total downlink traffic power. In this paper, example calculations of the CDF for the total downlink traffic power and power outage probabilities are provided as two applications for the developed model.

The following describe the organization for the rest of paper. Section II specifies the power-rate assignment framework and provides the solution. The proposed approximation and characterization of the cell site downlink traffic power are outlined in section III. Section IV presents numerical results obtained through both analysis and conventional Monte-Carlo simulations and provides discussion and comparison. Finally, concluding remarks are made at the end of this paper.

## II. BURST POWER ASSIGNMENT FRAMEWORK

Assume a general cellular configuration of a wireless CDMA system using an arbitrary frequency reuse factor. Let the service area of interest be denoted by cell 0, then the first tier of co-channel interferers contains 6 cells and are denoted by cell 1 to cell 6, while the second tier of interferers contains 12 cells and let these be denoted by cell 7 to cell 18. Interference beyond second tier is usually neglected relative to the interference caused by first and second tiers. Further let the CDMA system support $K$ arbitrary discrete data rates specified by $V = \{R_0, R_1, \ldots, R_{K-1}\}$. Focusing on the downlink resource management control, the cell site must manage the total cell site power and allocate the appropriate portions to provide a minimum acceptable signal quality for each supported connection. Referring to the assumed cellular configuration, the power transmitted from the cell site of interest, cell 0, and the interfering cells, cells 1 to 18, affect the downlink signal quality for a supported connection in the service area of interest. Let the total power budget of the cell site be equal to $P_T$ Watts. In addition, let the fraction allocated for overhead channels be denoted by $0<\beta<1$. This means the amount of power dedicated for overhead channels, $P_{ov}$, is equal to $\beta P_T$ Watts. Therefore, the maximum available traffic power is given by $(1-\beta)P_T$.

In typical CDMA systems, the downlink transmissions are not perfectly orthogonal at the mobile location. To account for the intracell interference, define the orthogonality factor $0<\rho<1$ such that $(1-\rho)$ represents the fraction of downlink intracell interference that degrades the signal quality. Therefore, for perfectly orthogonal downlink transmissions, the intracell interference is equal to zero. When the cell site of interest is serving $N$ simultaneous bursts, with the $i^{th}$ burst being at rate $r_i \in V$ for $i$ = 0, 1, …, $N$-1, then the implemented radio resource management procedure must allocate enough average forward link power, $P_i$, to support an optimal link quality. This is typically achieved by forcing the corresponding energy per-bit to noise spectral power density ratio, $(E_b/N_0)_i$, to be greater than a given minimum $E_b/N_0$ figure. Using, the above definitions, the $i^{th}$ link quality figure is given by

$$\left(\frac{E_b}{N_0}\right)_i = \frac{(W/r_i)P_i L_{i0}10^{\zeta_{i0}/10}}{(1-\rho)L_{i0}10^{\zeta_{i0}/10}\left[\sum_{l=0,l\neq i}^{N-1} P_l + Pov\right] + P_T\sum_{k=1}^{18} L_{ik}10^{\zeta_{ik}/10}} \qquad (1)$$

where $W$ is the system bandwidth, $L_{ik}$ and $\zeta_{ik}$ are the path loss coefficient and the shadowing factor, respectively, between the $i^{th}$ user in the cell of interest, and the $k^{th}$ cell site ($k$ = 0, 1, 2, …, 18). In this study we assume a path loss model where the received power is inversely proportional to distance raised to the path loss exponent, denoted by $\alpha$. The standard deviation of the shadowing factor $\zeta_{ik}$ is equal to $\sigma_{dB}$. It can be seen that the model in (1) accounts for both intracell interference, the first term in the denominator of (1), and intercell interference, the second term in denominator of (1). Furthermore, the model assumes conservatively that all co-channel cell sites are transmitting at their maximum power level, $P_T$. In (1), only large signal variations are considered where fast fading is assumed to be compensated for by multiple soft handoff legs, if supported, and fast power control.

The signal quality relation in (1) provides a system of $N$ linear equations in the downlink power allocations, $P_i$. This serves as the core relation employed for downlink call or burst admission control procedures for CDMA systems. The system can utilize (1) to determine whether the connection request at the specified bit rate, given the existing connections, can be supported or not. If a feasible solution for (1) can be found, then the assumed bit rate assignment for all connections $i$=0,1, …, $N$-1 can be supported. A feasible solution for (1) is a solution where $P_i \geq 0$ for $i$ = 0, 1, …, $N$-1, and the sum of all bursts power $\sum_{i=0}^{N-1} P_i$ is less or equal to the maximum possible traffic power $(1-\beta)P_T$ for the given bit rate assignments.

This paper attempts to characterize the sum of powers $\sum_{i=0}^{N-1} P_i$ allocated for users to provide the minimum acceptable downlink signal quality as a function of the co-channel interference, the radio frequency propagation model, and the shadowing process.

### *A. Sum Of Traffic Power Solution*

The quantity $\sum_{i=0}^{N-1} P_i$ is a function of not only the number of bursts and the corresponding service bit rates, but also the mobile locations and the radio frequency model. In this paper, we use (1) to derive a probabilistic model to characterize the sum of downlink traffic power to enable RRM studies accounting for the RF model and the soft capacity issue for CDMA multi-rate system. We further utilize this model to compute the probability of cell site power outage as a function of the number of supported bursts and the assumed bit rate assignments.

To solve for a feasible solution for a given bit rate assignment, if any, we can utilize (1) to set up a linear system of equations to solve for $P_i$, $i$=0, 1, …, $N$-1. Alternatively, the relation in (1) can be rewritten as

$$\left(\frac{E_b}{N_0}\right)_i = \frac{W}{r_i} \times \frac{P_i}{(1-\rho)\left[\sum_{l=0,l\neq i}^{N-1} P_l + Pov\right] + f_i P_T} \quad (2)$$

where $f_i$ is given by

$$f_i = \sum_{k=1}^{18} \left(L_{ik} 10^{\zeta_{ik}/10}\right) \Big/ \left(L_{i0} 10^{\zeta_{i0}/10}\right) \quad (3)$$

for $i$=0,1, …, $N$-1. Equation (2) can be rearranged as

$$P_i = G_i \left[\left(\sum_{i=0}^{N-1} P_i\right) + Pov + (1-\rho)^{-1} P_T f_i\right] \quad (4)$$

where $G_i = g_i/(1+g_i)$ and $g_i = (E_b/N_0)_i / (W/r_i)(1-\rho)$. Taking the sum for (4), and solving for the sum of traffic power, $\sum_{i=0}^{N-1} P_i$, yields

$$\sum_{i=0}^{N-1} P_i = P_T \frac{\beta \sum_{i=0}^{N-1} G_i + (1-\rho)^{-1} \sum_{i=0}^{N-1} G_i f_i}{1 - \sum_{i=0}^{N-1} G_i} \quad (5)$$

Substituting back in (4), the burst power assignment, $P_i$, should be given by

$$P_i = \frac{P_T \times G_i}{(1-\rho)} \left[\frac{\sum_{l=0}^{N-1} G_l f_l + \beta(1-\rho)}{1 - \sum_{l=0}^{N-1} G_l} + f_i\right] \quad (6)$$

As mentioned earlier, a set of $P_i$'s are feasible if and only if $\sum_{i=0}^{N-1} P_i \leq (1-\beta) P_T$. Using (5), the last constraint translates to

$$\sum_{i=0}^{N-1} G_i \left(1 + \frac{f_i}{1-\rho}\right) \leq (1-\beta) \quad (7)$$

This means, the feasibility of the solution $P_i$ is guaranteed if (7) holds for the assumed bit rates and signal quality thresholds (as represented by $G_i$'s), the subscriber locations and interference conditions (as represented by $f_i$'s), and the network parameters $\rho$ and $\beta$.

To derive a probabilistic model for the total traffic power specified by (5), it is required to characterize first the random variables $f_i$ and in turn the variable $\sum_{i=0}^{N-1} G_i f_i$. It is clear that terms other than $\sum_{i=0}^{N-1} G_i f_i$ are constants and depend only on the network parameters. This paper presents a novel approximate characterization for the random variables $f_i$ and $\sum_{i=0}^{N-1} G_i f_i$ that would lead to the desired characterization of the sum of downlink powers.

## III. Sum Of Traffic Power Characterization

### A. Lognormal Approximation for $f_i$ Parameter

The parameter $f_i$ defined in (3) is a function only of the location of the subscriber, the path loss model, and the shadowing process. Note that $f_i$ does not depend on the network parameters. Let the estimated mean and variance of ln($f_i$) be equal to $m_f$ and $\sigma_f^2$, respectively. The mean and variance of variable $f_i$ are a function of the path loss exponent $\alpha$ and shadowing process standard deviation $\sigma_{dB}$ defined in previous section. Performing Monte-Carlo simulations and studying the $f_i$ parameter, it is observed that the random variable $f_i$ is nearly lognormal for a wide and applicable range of $\alpha$ and $\sigma_{dB}$. Therefore, in this paper we approximate the distribution of the parameter $f_i$ by a lognormal distribution function specified by

$$f_{f_i}(x) = \frac{1}{\sqrt{2\pi}\sigma_f x} \exp\left[\frac{-(\ln x - m_f)^2}{2\sigma_f^2}\right] \quad (8)$$

where the parameters $m_f$ and $\sigma_f^2$ are the empirical mean and variance of the variable $f_i$, respectively. $\ln(x)$ is the natural logarithm function. In Fig. 1 and Fig. 2 we show the CDF for the random variable ln($f_i$) and the approximation given by (8) for $\sigma_{dB} = 6$ dB and $\sigma_{dB} = 12$ dB, and for different values of the path loss exponent $\alpha$, namely, 2, 4, and 6. The plots in Fig. 1 and Fig. 2 use normal probability paper so that the CDF for a pure lognormal random would appear as a straight line. It is clear from the plots that the approximation suggested by (8) is reasonably accurate for the used values of $\sigma_{dB}$ and $\alpha$. The figures show the corresponding CDFs on a range from $10^{-6}$ to 1-$10^{-6}$ as a function of $f_i$ in decibels. The approximation by the lognormal distribution improves as the standard deviation of the shadowing process $\sigma_{dB}$, increases. For a given value $\sigma_{dB}$, the approximation is better in the case for smaller values of the path loss exponent $\alpha$.

As alluded to earlier, the parameters $m_f$ and $\sigma_f^2$ are function of the environment and the RF propagation model only. Therefore, they can be made available for a wide range of environments and RF models. For example, Table I provides the empirical mean and standard deviations for ln($f_i$) for a conventional hexagonal cellular service area where the cell of interest is surrounded by 18 co-channel cells. The assumed RF propagation model is the simple path loss exponent model where the signal attenuation is inversely proportional to the distance raised to the exponent $\alpha$.

### B. Distribution Function for Sum of Traffic Power

To compute the PDF for the cell site downlink traffic power, $\sum_{i=0}^{N-1} P_i$, the relation in (5) is utilized. Let the random variable $\sum_{i=0}^{N-1} P_i$ be denoted by $Z$ while the random variable $\sum_{i=0}^{N-1} G_i f_i$ be denoted by $Y$. It is clear that $Z$ is a linear transformation of the random variable $Y$. That is, $Z = aY + b$, where the constants $a$ and $b$ are given by $P_T \Big/ \left[\left(1\text{-}\sum_{i=0}^{N-1} G_i\right)(1\text{-}\rho)\right]$ and $P_T \beta \sum_{i=0}^{N-1} G_i \Big/ (1\text{-}\sum_{i=0}^{N-1} G_i)$, respectively. Therefore, the desired PDF for $Z$ can be written as

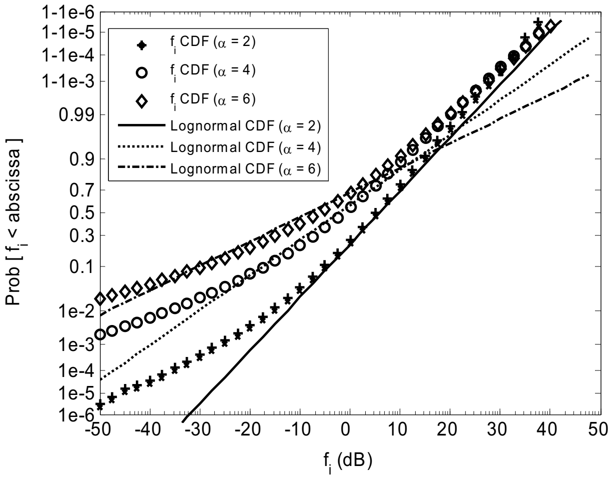


Figure 1. The CDF of the parameter $f_i$ for $\sigma_{dB}$ = 6 dB compared with lognormal CDF plotted on normal probability paper.

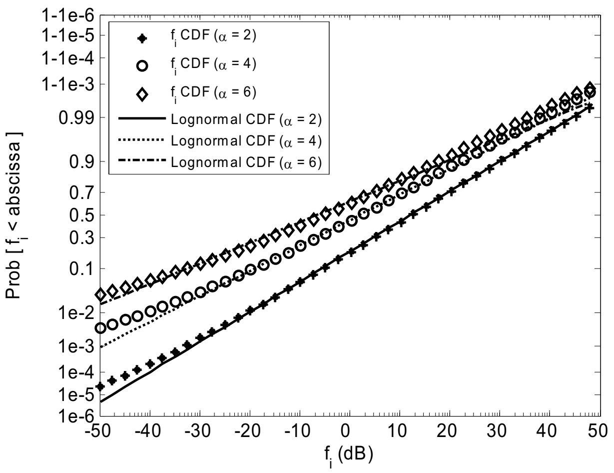


Figure 2. The CDF of the parameter $f_i$ for $\sigma_{dB}$ = 12 dB compared with lognormal CDF plotted on normal probability paper.

TABLE I. MEAN AND STANDARD DEVIATION FOR THE VARIABLE LN($f_i$) FOR DIFFERENT RF AND SHADOWING PARAMETERS.

| **Shadowing ($\sigma_{dB}$)** | **Mean of ln($f_i$), $m_f$** | | | **Standard deviation of ln($f_i$), $\sigma_f$** | | |
|---|---|---|---|---|---|---|
| | ***α* = 2** | ***α* = 4** | ***α* = 6** | ***α* = 2** | ***α* = 4** | ***α* = 6** |
| **4 dB** | 0.62 | -1.32 | -2.85 | 1.45 | 2.54 | 3.83 |
| **6 dB** | 1.04 | -1.00 | -2.60 | 1.83 | 2.77 | 3.96 |
| **8 dB** | 1.57 | -0.57 | -2.28 | 2.26 | 3.06 | 4.16 |
| **10 dB** | 2.19 | -0.05 | -1.88 | 2.73 | 3.41 | 4.41 |
| **12 dB** | 2.86 | 0.54 | -1.39 | 3.22 | 3.81 | 4.70 |
| **14 dB** | 3.57 | 1.18 | -0.84 | 3.72 | 4.24 | 5.03 |

$$f_Z(z)=\frac{1}{a}f_Y\left(\frac{z-b}{a}\right) \tag{9}$$

where $f_Y(y)$ is the PDF for the variable $Y$. Equivalently, we can also compute the CDF for the random variable $Z$ to be

$$F_Z(z)=F_Y\left(\frac{z-b}{a}\right) \tag{10}$$

where $F_Y(y)$ is the CDF for the variable $Y$.

In the following we highlight the calculations leading to the characterization of the random variable $Y$. Using the approximation suggested in this paper, the random variable $Y$ represents the sum of $N$ independent lognormal random variables where each variable is given by $G_i f_i$. Scaling by the constant $G_i$ makes the random variables non identically distributed, since $G_i$ depends on the assigned bit rate and the $E_b/N_0$ threshold target of the burst. While there is a significant amount of work on finding the sum of independent lognormal random variables in the literature, such as [10][11] and the references therein, we utilize the conventional approach. That is we calculate the characteristic function for our random variable, perform the multiplication in the frequency domain to obtain the overall characteristic function, and then invert the result to obtain the required PDF or CDF. However, the calculation of the characteristic function for the lognormal random variable is in itself a problematic issue since the transformation involves a highly oscillatory integrand. Recently two new techniques were developed in [12] and [13] to facilitate the calculation of the characteristic function of a lognormal random variable. In this paper, we utilize the efficient method outlined in [12] to compute the characteristic function of lognormal variable $f_i$ whose parameters $m_f$ and $\sigma_f^2$ are specified in Table I. Therefore, the characteristic function of the random variable $Y$ is given by

$$\Theta_Y(\omega)=\prod_{i=0}^{N-1}\Theta_{G_i f_i}(\omega)=\prod_{i=0}^{N-1}\Theta_f(G_i\omega) \tag{11}$$

where the characteristic function of $f_i$, $\Theta_f(\omega)$, need to be evaluated only once. The CDF of $Y$ is then calculated using

$$F_Y(y)=\frac{2}{\pi}\int_0^\infty \mathrm{Re}\{\Theta_Y(\omega)\}\sin(\omega y)/\omega\, d\omega \tag{12}$$

where $\mathrm{Re}\{x\}$ is the real part of $x$. The relation in (12) exploits the fact that the corresponding PDF or CDF is real-valued and is zero for negative values of the abscissa [14]. The integrand in (12) is again highly oscillatory and the modified Clenshaw-Curtis quadrature is used to numerically evaluate the integral [15][16].

Finally, to evaluate the system power outage probabilities we can utilize (10) to calculate the probability of outage by calculating $1-F_Z(z)$ evaluated at $z$ equal to the total cell site power, $P_T$. Alternatively, we can directly utilize (7) and (12) as follows. The relation in (7) can be rewritten in terms of the random variable $Y$ as

$$Y=\sum_{i=0}^{N-1}G_i f_i \le \left[(1-\beta)-\sum_{i=0}^{N-1}G_i\right](1-\rho) \tag{13}$$

Hence, the power outage probability is given in terms of the CDF for the random variable $Y$ by

$$\text{Prob}\left[\text{power outage}\right]=1-F_Y\left(\gamma\right) \quad (14)$$

where $\gamma$ is given by $\left[\left(1-\beta\right)-\sum_{i=0}^{N-1}G_i\right]\left(1-\rho\right)$.

The relations specified by (10) or equivalently by (12), and that of (14), represent the main contribution of this paper. The following section utilizes these equations to present numerical examples and provide comparisons with results obtained using conventional Monte-Carlo simulations.

## IV. Numerical Example

In this section we present numerical examples demonstrating the accuracy of the proposed approximation for characterizing the sum of downlink traffic power in a CDMA network. While the results shown below assume specific CDMA network parameters, the developed approach is identical for all CDMA data networks. Assume a cdma2000 (1xRTT) network [17] supporting multiple data rates specified by the set $V=\left\{R_j=2^j R_0, j=0,1,\cdots,K-1\right\}$ where $R_0$ = 9.6 kb/s and $K$ = 5. Furthermore, assume the cell site maximum transmit power, $P_T$, is equal to 24 Watts with the overhead fraction for non-traffic channels, $\beta$, set to 0.2 of the total power budget. We also assume the that forward link channels are not perfectly orthogonal and the orthogonality parameter, $\rho$, is equal to 10%. Finally, we assume the target link quality figure $E_b/N_0$ is equal to 10 dB while the path loss exponent used for the RF propagation model, $\alpha$, is equal to 4. For the results shown in this section, we test the model for the two extreme values of $\sigma_{\text{dB}}$ of 6 dB and 12 dB as the standard deviation for the shadowing process.

For a given number of users, $N$, served by the basestation, let the vector $\overline{n}=\left(n_0,n_1,\cdots,n_{K-1}\right)$ represent the system state. In this notation, $n_j$ is the number of users assigned to the $j^{\text{th}}$ system data rate $R_j$, for $j$=0, 1, …, $K$-1. It follows that $\sum_{j=0}^{K-1} n_j=N$. For each iteration of the Monte-Carlo simulations, the number of users specified by $\overline{n}$ are located randomly in the cell of interest corresponding to a particular vector of $\overline{f_i}=\left(f_{i,0},f_{i,1},\cdots,f_{i,N-1}\right)$, where every user location corresponds to some random value of $f_i$ calculated by (3). The power required to support these users, referred to by $Z$ in the previous section, is calculated using (5) and we also calculate the corresponding value of the random variable $Y$ as specified by (13). If the required power to support the specific $\overline{n}$ is greater than $(1-\beta)P_T$, an outage event is recorded as defined by [6]. The iteration is repeated $10^5$ times to average overall possible locations in the cell of interest and the shadowing process.

Results shown in Fig. 3 and Fig. 4 depict the CDF for the random variable $Y$ which is a scaled version of the random variable representing the downlink traffic power. The results are shown for the case of $\sigma_{\text{dB}}$ equal to 6 dB in Fig. 3, and for the case of $\sigma_{\text{dB}}$ equal to 12 dB in Fig. 4. In these curves, a particular combination of users and bit rate assignment specified by $\overline{n}$ is used as shown in the legends of the graphs. A comparison between the CDF obtained using Monte-Carlo simulations and that obtained using the developed relation in (12) clearly indicates that the assumed lognormal approximation for the parameter $f_i$ is appropriate. Even for cases where $\sigma_{\text{dB}}$ is as low as 6 dB or where the path loss exponent $\alpha$ is as high as 4 or 6, the calculations still accurately match the empirical CDF for $Y$. For the used combinations, and at any arbitrary value of $f_i$, the CDF for $Y$ has a value that increases as the user combination is changed in the following order: (8, 0, 0, 0, 0), (3, 0, 0, 1, 0), (1, 2, 0, 0, 0), and (0, 1, 1, 0, 0).

To test for progressively increasing number of users assigned to the $j^{\text{th}}$ system rate, $R_j$, we assume a user combination specified by $\overline{n}=\left(n_0,n_1,\cdots,n_{K-1}\right)$ where $n_i$'s for $i\neq j$ are given and $n_j$, denoted by $X$, is allowed to vary from 0 to some upper limit equal to $n_{j,\max}$. The upper limit may be determined again by the constraint outlined in (7).

For each user combination of the form $\overline{n}=\left(n_0,n_1,\ldots,n_{j-1},X,n_{j+1},\ldots,n_{K-1}\right)$ where $X$ is equal to 0, 1, …, $n_{j,\max}$, the outage probability is evaluated using Monte-Carlo simulation as outlined earlier in this section and compared against the value obtained using (14).

The outage probability curves are shown in Fig. 5 and Fig. 6 for $\sigma_{\text{dB}}$ equal to 6 dB and 12 dB, respectively. The rest of the network parameters are same as those used in previous figures. The user combinations considered in Fig. 5 and Fig. 6 are ($X$,0,0,0,0), (3,0,$X$,1,0), ($X$,1,1,0,0), and ($X$,2,0,0,0) where $X$ is the range of possible number of users for the corresponding bit rate assignment. Again, we observe that the empirical outage probability figures are closely matched by those resulting from the developed model in this paper. In other words, the model suggested in this paper for the PDF for the downlink traffic and the underlying lognormal approximation for the parameter $f_i$, approximate very well the results obtained using Monte-Carlo simulations. The outage curves also depict the effect of high bit rate assignments on the total downlink traffic power budget. As the system uses high bit rates such as in the case of (3, 0, $X$, 1, 0), the outage probability sharply increases relative to the cases where only low bit rates are used such as the cases of ($X$, 0, 0, 0, 0) or ($X$, 2, 0, 0, 0).

## V. Conclusions

In this paper we develop a model for characterizing the sum of downlink traffic power for a general direct-sequence CDMA system supporting multi-rate data bursts. In specific, we present a formula for approximating the cumulative probability distribution function of the cell site traffic power as a function of the supported bit rates, the target link quality figure, and the other network parameters.

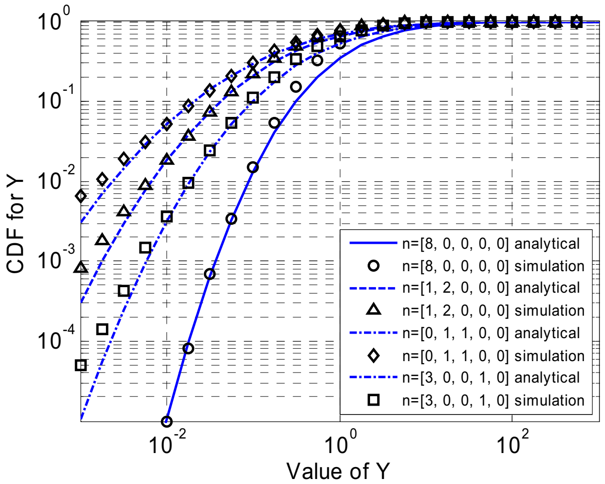


Figure 3. Cumulative probability distribution function for the random variable $Y$ ($K$=5, $\alpha$ = 4, and $\sigma_{dB}$=6 dB, $E_b/N_0$ = 10 dB).

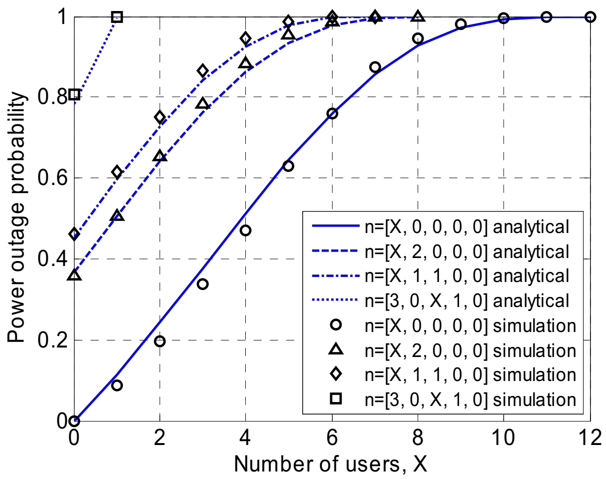


Figure 5. Cell site power outage probability as a function of the number of ongoing bursts ($K$=5, $\alpha$ = 4, and $\sigma_{dB}$=6 dB, $E_b/N_0$ = 10 dB).

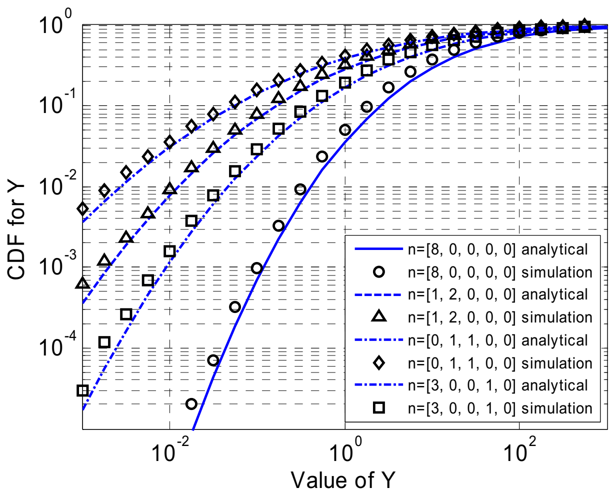


Figure 4. Cumulative probability distribution function for the random variable $Y$ ($K$=5, $\alpha$ = 4, and $\sigma_{dB}$=12 dB, $E_b/N_0$ = 10 dB).

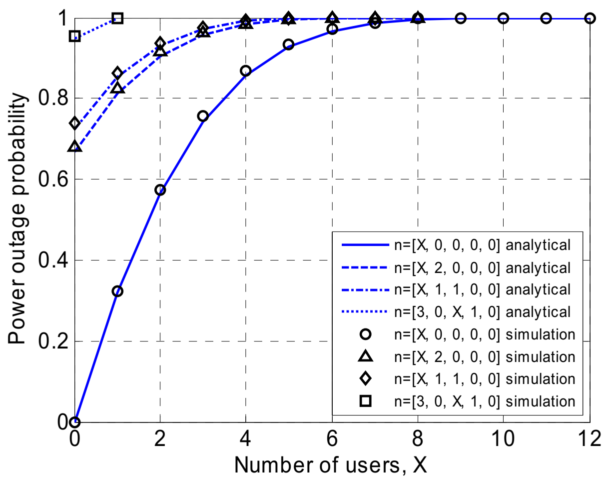


Figure 6. Cell site power outage probability as a function of the number of ongoing bursts ($K$=5, $\alpha$ = 4, and $\sigma_{dB}$=12 dB, $E_b/N_0$ = 10 dB).

The method relies on approximating the empirical distribution of the parameter $f_i$, which is specific to a particular radio frequency propagation model and shadowing process, by a lognormal random variable. The approximating lognormal distribution is selected such that its mean and standard deviation are equal to the empirical mean and standard deviation of the parameter $f_i$. The accuracy of the approximation is appropriate for a wide range of applicable values of the path loss exponent and the standard deviation of the shadowing process. As an application, the paper shows how this approximation can be utilized to compute the probability density function of the cell site traffic power and the probability of forward link power outage for several examples of data bit rate assignments. Results indicate the developed model provide accurate calculations of the traffic power distribution and the outage probabilities when compared to conventional Monte-Carlo simulations.

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