999KING FAHD UNIVERSITY OF PETROLEUM \& MINER ALS
COLLEGE OF COMPUTER SCIENCES \& ENGINEERING
COMPUTER ENGINEERING DEPARTMENT
COE-587 - Computer System Performance Evaluation December 10 ${ }^{\text {th }}, 2009$ - Major Exam

## Student Name:

Student Number:
Exam Time: 90 mins

- Do not open the exam book until instructed
- Answer all questions
- All steps must be shown
- Any assumptions made must be clearly stated

| Question No. | Max Points |  |
| :---: | :---: | :--- |
| 1 | 20 |  |
| 2 | 20 |  |
| 3 | 20 |  |
| 4 | 30 |  |
| Total: |  |  |

Q.1) ( 20 points) On the subject of selection of techniques and metrics
a) ( $\mathbf{1 0}$ points) There are three techniques for performance evaluation. These techniques differ in terms of the criteria listed in the table shown below. On the table, name the three evaluation techniques, and complete the rest of the table by characterizing each of the techniques with respect to the listed criteria.

Table: Criteria for selecting an evaluation technique.

| Criterion/Technique | (1) | (2) | (3) |
| :--- | :--- | :--- | :--- |
| 1. Stage |  |  |  |
| 2. Time required |  |  |  |
| 3. Tools |  |  |  |
| 4. Accuracy |  |  |  |
| 5. Trade-off <br> evaluation |  |  |  |
| 6. Cost |  |  |  |
| 7. Saleability |  |  |  |

b) ( 5 points) Draw an ideal throughput versus load curve.
c) (3 points) Define the following terms: nominal capacity, usable capacity, and knee capacity.
d) ( $\mathbf{2}$ points) Give an example where a high is better for a performance metric? Give an example where a low is better for a performance metric?
Q.2) ( 20 points) On the subject of the art of workload selection and workload characterization
a) The best way to start the workload selection is to view the system as a service provider. Numerating the services provided by the system is the first step in a systematic performance evaluation study. In this context
a.1) (4 points) Define the terms: System Under Test (SUT) and Component Under Study (CUS).
a.2) ( 6 points) Consider the Domain Name System (DNS) for the Internet example used in the class. For ONE instance of the system, it is required to provide specification for the SUT in terms of (a) services, (b) factors, (c) metrics, and (d) workload.
b) On the Principle component Analysis (PCA) subject
b.1) (3 points) What is the purpose of the PCA procedure?
b.2) ( $\mathbf{4}$ points) List the main steps involved in performing the PCA.
b.3) ( $\mathbf{3}$ points) For the case of two variables: $x_{s}$ and $x_{r}$, how is the correlation matrix computed?
Q.3) (20 points) On the subject of summarizing measured data:

Assume that response times for the DNS considered in class are normally distributed with a mean of 10 seconds and a standard deviation of 2 second. Determine the following:
a) ( $\mathbf{2}$ points) What is the probability of the response time being more than 12 seconds?
b) ( 2 points) What is the probability of the response time being less than 8 seconds?
c) ( $\mathbf{2}$ points) What percentage of responses will take between 8 and 12 seconds?
d) ( 2 points) What is the 90 -percentile execution time?
e) ( 2 points) Sketch the probability density function for the response times.
f) (2 points) Sketch the cumulative probability function for the response times.
g) ( 2 points) Is the random variable (i.e. the response time) continuous or discrete?
h) (2 points) Compute the coefficient of variation for the response times.
i) (4 points) List 4 different indices of data dispersion.
Q.4) (30 points) On the subject of confidence interval and its applications, answer the following questions:
a) ( $\mathbf{1 5}$ points) The difference in the processor times of two different implementations of the same algorithm was measured on seven similar workload. The differences are $\{1.5,2.6,-1.8,1.3,-0.5,1.7$, 2.4 . Can we say with $80 \%$ confidence that one implementation is superior to the other?
b) ( 5 points) To compute the confidence intervals we utilize quantiles either from the normal distribution or the student $t$ distribution. What are conditions under which each type of quantile is used?
c) ( $\mathbf{1 0}$ points) Explain briefly the "visual" test that can be performed to compare performance figures for two systems? Is it used for paired or unpaired observations?

Tables/Formulas you might need:

## A. 1 AREA OF THE UNIT NORMAL DISTRIBUTION

Table A. 1 lists area between 0 and $z$. For example, the area between $z=0$ and $z=1.03$ is 0.3485 . Due to symmetry of the normal distribution, the area between $z=0$ and $z=-1.03$ is also the same.

table A. 1 Area of the Unit Normal Distribution

| $\boldsymbol{z}$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2703 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4915 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4377 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.46333 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |

stansmeal tables

## A. 2 QUANTITIES OF THE UNIT NORMAL DISTRIBUTION

Table A. 2 lists $z_{p}$ for a given $p$. For example, for a two-sided confidence interval at $95 \%, \alpha=0.05$ and $p=1-\alpha / 2=0.975$. The entry in the row labeled 0.97 and column labeled 0.005 gives $z_{p}=1.960$.


TABLE A. 2 Quantiles of the Unit Normal Distribution

| $p$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.5 | 0.000 | 0.025 | 0.050 | 0.075 | 0.100 | 0.126 | 0.151 | 0.176 | 0.202 | 0.228 |
| 0.6 | 0.253 | 0.279 | 0.305 | 0.332 | 0.358 | 0.385 | 0.412 | 0.440 | 0.468 | 0.496 |
| 0.7 | 0.524 | 0.553 | 0.583 | 0.613 | 0.643 | 0.674 | 0.706 | 0.739 | 0.772 | 0.806 |
| 0.8 | 0.842 | 0.878 | 0.915 | 0.954 | 0.994 | 1.036 | 1.080 | 1.126 | 1.175 | 1.227 |
|  |  |  |  |  |  |  |  |  |  |  |
| $p$ | 0.000 | 0.001 | 0.002 | 0.003 | 0.004 | 0.005 | 0.006 | 0.007 | 0.008 | 0.009 |
| 0.90 | 1.282 | 1.287 | 1.293 | 1.299 | 1.305 | 1.311 | 1.317 | 1.323 | 1.329 | 1.335 |
| 0.91 | 1.341 | 1.347 | 1.353 | 1.359 | 1366 | 1.372 | 1.379 | 1.385 | 1.392 | 1.398 |
| 0.92 | 1.405 | 1.412 | 1.419 | 1.426 | 1.433 | 1.440 | 1.447 | 1.454 | 1.461 | 1.468 |
| 0.93 | 1.476 | 1.483 | 1.491 | 1.499 | 1.506 | 1.514 | 1.522 | 1.530 | 1.538 | 1.546 |
| 0.94 | 1.555 | 1.563 | 1.572 | 1.580 | 1.589 | 1.598 | 1.607 | 1.616 | 1.626 | 1.635 |
| 0.95 | 1.645 | 1.655 | 1.665 | 1.675 | 1.685 | 1.695 | 1.706 | 1.717 | 1.728 | 1.739 |
| 0.96 | 1.751 | 1.762 | 1.774 | 1.787 | 1.799 | 1.812 | 1.825 | 1.838 | 1.852 | 1.866 |
| 0.97 | 1.881 | 1.896 | 1.911 | 1.927 | 1.943 | 1.960 | 1.977 | 1.995 | 2.014 | 2.034 |
| 0.98 | 2.054 | 2.075 | 2.097 | 2.120 | 2.144 | 2.170 | 2.197 | 2.226 | 2.257 | 2.290 |


| $\boldsymbol{P}$ | 0.0000 | 0.0001 | 0.0002 | 0.0003 | 0.0004 | 0.0005 | 0.0006 | 0.0007 | 0.0008 | 0.0009 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.990 | 2.326 | 2.330 | 2.334 | 2.338 | 2.342 | 2.346 | 2.349 | 2.353 | 2.357 | 2.362 |
| 0.991 | 2.366 | 2.370 | 2.374 | 2.378 | 2.382 | 2.387 | 2.391 | 2.395 | 2.400 | 2.404 |
| 0.992 | 2.409 | 2.414 | 2.418 | 2.423 | 2.428 | 2.432 | 2.437 | 2.442 | 2.447 | 2.452 |
| 0.993 | 2.457 | 2.462 | 2.468 | 2.473 | 2.478 | 2.484 | 2.489 | 2.495 | 2.501 | 2.506 |
| 0.994 | 2.512 | 2.518 | 2.524 | 2.530 | 2.536 | 2.543 | 2.549 | 2.556 | 2.562 | 2.569 |
| 0.995 | 2.576 | 2.583 | 2.590 | 2.597 | 2.605 | 2.612 | 2.620 | 2.628 | 2.636 | 2.644 |
| 0.996 | 2.652 | 2.661 | 2.669 | 2.678 | 2.687 | 2.697 | 2.706 | 2.716 | 2.727 | 2.737 |
| 0.997 | 2.748 | 2.759 | 2.770 | 2.782 | 2.794 | 2.807 | 2.820 | 2.834 | 2.848 | 2.863 |
| 0.998 | 2.878 | 2.894 | 2.911 | 2.929 | 2.948 | 2.968 | 2.989 | 3.011 | 3.036 | 3.062 |
| 0.999 | 3.090 | 3.121 | 3.156 | 3.195 | 3.239 | 3.291 | 3.353 | 3.432 | 3.540 | 3.719 |

## A. 4 QUANTILES OF THE $t$ DISTRIBUTION

Table A. 4 lists $t_{[p ; \pi]}$. For example, the $t_{[0.95 ; 13]}$ required for a two-sided $90 \%$ confidence interval of the mean of a sample of 14 observation is 1.771 .


TABLE A. 4 Quantiles of the $t$ Distribution

|  | $p$ |  |  |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $n$ | 0.6000 | 0.7000 | 0.8000 | 0.9000 | 0.9500 | 0.9750 | 0.9950 | 0.9995 |
| 1 | 0.325 | 0.727 | 1.377 | 3.078 | 6.314 | 12.706 | 63.657 | 636.619 |
| 2 | 0.289 | 0.617 | 1.061 | 1.886 | 2.920 | 4.303 | 9.925 | 31.599 |
| 3 | 0.277 | 0.584 | 0.978 | 1.638 | 2.353 | 3.182 | 5.841 | 12.924 |
| 4 | 0.271 | 0.569 | 0.941 | 1.533 | 2.132 | 2.776 | 4.604 | 8.610 |
| 5 | 0.267 | 0.559 | 0.920 | 1.476 | 2.015 | 2.571 | 4.032 | 6.869 |
| 6 | 0.265 | 0.553 | 0.906 | 1.440 | 1.943 | 2.447 | 3.707 | 5.959 |
| 7 | 0.263 | 0.549 | 0.896 | 1.415 | 1.895 | 2.365 | 3.499 | 5.408 |
| 8 | 0.262 | 0.546 | 0.889 | 1.397 | 1.860 | 2.306 | 3.355 | 5.041 |
| 9 | 0.261 | 0.543 | 0.883 | 1.383 | 1.833 | 2.262 | 3.250 | 4.781 |
| 10 | 0.260 | 0.542 | 0.879 | 1.372 | 1.812 | 2.228 | 3.169 | 4.587 |
| 11 | 0.260 | 0.540 | 0.876 | 1.363 | 1.796 | 2.201 | 3.106 | 4.437 |
| 12 | 0.259 | 0.539 | 0.873 | 1.356 | 1.782 | 2.179 | 3.055 | 4.318 |
| 13 | 0.259 | 0.538 | 0.870 | 1.350 | 1.771 | 2.160 | 3.012 | 4.221 |
| 14 | 0.258 | 0.537 | 0.868 | 1.345 | 1.761 | 2.145 | 2.977 | 4.140 |
| 15 | 0.258 | 0.536 | 0.866 | 1.341 | 1.753 | 2.131 | 2.947 | 4.073 |
| 16 | 0.258 | 0.535 | 0.865 | 1.337 | 1.746 | 2.120 | 2.921 | 4.015 |
| 17 | 0.257 | 0.534 | 0.863 | 1.333 | 1.740 | 2.110 | 2.898 | 3.965 |
| 18 | 0.257 | 0.534 | 0.862 | 1.330 | 1.734 | 2.101 | 2.878 | 3.922 |
| 19 | 0.257 | 0.533 | 0.861 | 1.328 | 1.729 | 2.093 | 2.861 | 3.883 |
| 20 | 0.257 | 0.533 | 0.860 | 1.325 | 1.725 | 2.086 | 2.845 | 3.850 |
| 21 | 0.257 | 0.532 | 0.859 | 1.323 | 1.721 | 2.080 | 2.831 | 3.819 |
| 22 | 0.256 | 0.532 | 0.858 | 1.321 | 1.717 | 2.074 | 2.819 | 3.792 |
| 23 | 0.256 | 0.532 | 0.858 | 1.319 | 1.714 | 2.069 | 2.807 | 3.768 |
| 24 | 0.256 | 0.531 | 0.857 | 1.318 | 1.711 | 2.064 | 2.797 | 3.745 |
| 25 | 0.256 | 0.531 | 0.856 | 1.316 | 1.708 | 2.060 | 2.787 | 3.725 |
| 26 | 0.256 | 0.531 | 0.856 | 1.315 | 1.706 | 2.056 | 2.779 | 3.707 |
| 27 | 0.256 | 0.531 | 0.855 | 1.314 | 1.703 | 2.052 | 2.771 | 3.690 |
| 28 | 0.256 | 0.530 | 0.855 | 1.313 | 1.701 | 2.048 | 2.763 | 3.674 |
| 29 | 0.256 | 0.530 | 0.854 | 1.311 | 1.699 | 2.045 | 2.756 | 3.659 |
| 30 | 0.256 | 0.530 | 0.854 | 1.310 | 1.697 | 2.042 | 2.750 | 3.646 |
| 60 | 0.254 | 0.527 | 0.848 | 1.296 | 1.671 | 2.000 | 2.660 | 3.460 |
| 90 | 0.254 | 0.526 | 0.846 | 1.291 | 1.662 | 1.987 | 2.632 | 3.402 |
| 120 | 0.254 | 0.526 | 0.845 | 1.289 | 1.658 | 1.980 | 2.617 | 3.373 |

## Box 12.1 Summarizing Observations

Given: A sample $\left\{x_{1}, x_{2}, \ldots, x_{n}\right\}$ of $n$ observations.

1. Sample arithmetic mean: $\bar{x}=\frac{1}{n} \sum_{i=1}^{n} x_{i}$
2. Sample geometric mean: $\dot{x}=\left(\prod_{i=1}^{n} x_{i}\right)_{n}^{1 / n}$
3. Sample harmonic mean: $\ddot{x}=\frac{n}{\frac{1}{x_{1}}+\frac{1}{x_{2}}+\cdots+\frac{1}{x_{n}}}$
4. Sample median: $\begin{cases}x_{((n-1) / 2)} & \text { if } n \text { is odd } \\ 0.5\left(x_{(n / 2)}+x_{((1+n) / 2)}\right) & \text { otherwise }\end{cases}$

Here $x_{(i)}$ is the $i$ th observation in the sorted set.
5. Sample mode = observation with the highest frequency (for categorical data).
6. Sample variance: $s^{2}=\frac{1}{n-1} \sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}$
7. Sample standard deviation: $s=\sqrt{\frac{1}{n-1} \sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}$
8. Coefficient of variation $=s / \bar{x}$
9. Coefficient of skewness $=\frac{1}{n s^{3}} \sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{3}$
10. Range: Specify the minimum and maximum.
11. Percentiles: $100 p$-percentile $x_{p}=x_{(\lfloor 1+(n-1) p\rfloor)}$.
12. Semi-interquartile range $\mathrm{SIQR}=\frac{Q_{3}-Q_{1}}{2}=\frac{x_{0.75}-x_{0.25}}{2}$
13. Mean absolute deviation $=\frac{1}{n} \sum_{i=1}^{n}\left|x_{i}-\bar{x}\right|$

## Box 13.1 Confidence Intervals

1. Given: A sample $\left\{x_{1}, x_{2}, \ldots, x_{n}\right\}$ of $n$ observations. $\bar{x}=$ sample mean; $s=$ sample standard deviation
(a) Standard error of the sample mean: $\sigma_{\bar{Y}}=\frac{s}{\sqrt{n}}$
(b) $100(1-\alpha) \%$ two-sided confidence interval for the mean: $\bar{x} \mp z_{1-\alpha / 2} s / \sqrt{n}$
If $n \leq 30^{\dagger}: \bar{x} \mp t_{[1-\alpha / 2 ; n-1)^{5}} / \sqrt{n}$
(c) $100(1-\alpha) \%$ one-sided confidence interval for the mean: $\left(\bar{x}, \bar{x}+z_{1-\alpha} s / \sqrt{n}\right)$ or $\left(\bar{x}-z_{1-\alpha} s / \sqrt{n}, \bar{x}\right)$

$$
\text { If } n \leq 30^{\dagger}:\left(\bar{x}, \bar{x}+t_{\left.[1-\alpha ; n-1)^{s} / \sqrt{n}\right) \text { or }\left(\bar{x}-t_{[1-\alpha ;-1]^{s}}^{s} / \sqrt{n}, \bar{x}\right)}\right.
$$

2. To compare two systems using unpaired observations:
(a) The standard error of the mean difference: $s=\sqrt{\frac{s_{a}^{2}}{n_{a}}+\frac{s_{b}^{2}}{n_{b}}}$
(b) The effective number of dcgrecs of freedom:

$$
\nu=\frac{\left(s_{a}^{2} / n_{a}+s_{b}^{2} / n_{b}\right)^{2}}{\frac{1}{n_{a}+1}\left(\frac{s_{a}^{2}}{n_{a}}\right)^{2}+\frac{1}{n_{b}+1}\left(\frac{s_{b}^{2}}{n_{b}}\right)^{2}}-2
$$

(c) The confidence interval for the mean difference: $\left(\bar{x}_{a}-\bar{x}_{b}\right) \mp t_{\left[1-\alpha / 2 ;\left.\nu\right|^{s}\right.}$
3. If $n_{1}$ of the $n$ observations belong to a certain class, the following statistics can be reported for the class:
(a) Proportion of the observations in the class: $p=\frac{n_{1}}{n}$
(b) $100(1-\alpha) \%$ two-sided confidence interval for the proportion ${ }^{\text {: }}$ : $p \mp z_{1-\alpha / 2} \sqrt{\frac{p(1-p)}{n}}$
(c) $100(1-\alpha) \%$ one-sided confidence interval for the proportion ${ }^{2}$ :

$$
\left(p, p+z_{1-a} \sqrt{\frac{p(1-p)}{n}}\right) \quad \text { or } \quad\left(p-z_{1-a} \sqrt{\frac{p(1-p)}{n}}, p\right)
$$

${ }^{\dagger}$ Only for samples from normal populations.
${ }^{\text {t }}$ Provided $n p \geq 10$.

