Example 3.4 (page 94):

Channel between 4 MHz and 3 MHz – SNR = 24 dB

Capacity?

Solution:

Shannon – SNR = 24 dB = 10^(24/10) = 251

Bandwidth B = 4 – 3 = 1 MHz = 1x10^6 Hz

$$C = B\log_2(1 + SNR) = 10^6 \log_2(1 + 251) \approx 8\ \text{Mbps}$$

This theoretical limit is probably unlikely to be reached. However, assuming we can reach this limit, what would be the minimum signaling levels (i.e. size of alphabets for symbols, M) needed?

$$C = 2B\log_2 M \rightarrow 8 \times 10^6 = 2(10^6)\log_2 M$$

Therefore, $\log_2 M = 4$ or $M = 16$.

Problem 3.7 (page 98):

Find the period of the function $f(t) = (10\cos t)^2$.

Solution:

VERY IMPORTANT CONCEPT

$$f(t) = (10\cos t)^2 = 100[\cos t]^2 = 100\left[\frac{1+\cos(2t)}{2}\right] = 50 + 50\cos(2t)$$

Therefore, one can use $\frac{2\pi}{T}t = 2t$ ➔ $T = \pi$ time units.

Very interesting questions:

- What is the power for the periodic signal $f(t)$?
- What is the corresponding Fourier Series Expansion (F.S.E)?
- Compute/Plot the corresponding PSD?

The **key observation** is that the periodic signal is ALREADY given in terms of the BASIC COMPONENTS (DC + HARMONICS) – therefore, no need to go through the standard procedure and evaluate integrals to obtain the A0, An, and Bn coefficients.

Comparing the expression for the F.S.E

$$f(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos(2\pi n f_0 t) + \sum_{n=1}^{\infty} B_n \sin(2\pi n f_0 t)$$

With the above expression for

$$f(t) = 50 + 50\cos(2t) = 50 + 50\cos\left(2\pi\left(\frac{1}{\pi}\right)t\right)$$

This leads to the following:

$\frac{A_0}{2} = 50$ or $A_0 = 100$. Note the DC component is equal to 50 Volts.

Furthermore, $T = \pi = f_0^{-1}$, or $f_0 = \frac{1}{\pi}$ Hz.

$A_1 = 50$ while $A_n = 0$ for $n = 2,3, \ldots$. Finally, all Bn's are equal to zeros.

Power for $f(t)$ is simply $50^2 + \frac{(50)^2}{2} = 3750$ Watts.

The PSD function is given by:

$$PSD(f) = \begin{cases} 50^2 & f = 0 \text{ Hz} \\ \dfrac{50^2}{2} & f = f_0 \end{cases}$$

Note the PSD function does not exist for other multiples of f_0.

Problem 3.13 (page 98) – Previous Exam problem:

Suppose that a black-and-white digitized TV picture is to be transmitted from a source that uses a matrix of 480X500 picture elements (pixels), where each pixel can take on one of 32 *intensity* values. Assume that 30 pictures (or frames) are sent per second.

a) (5 points) What is the source rate in pixels/second?

b) (5 points) What is the source rate R in bits/sec?

c) (5 points) Discuss how the parameters could be modified to allow transmission of *color* TV signals without increasing the required value for R?

d) (10 points) Assume that the TV picture is to be transmitted over a channel with 4.5 MHz bandwidth and a 35 dB signal-to-noise ratio. Find the capacity of the channel in bits per second.

e) (5 points) Calculate the spectral efficiency of the channel specified in part (d). Specify the units.

Solution:

a) (30 pictures/s) (480 x 500 pixels/picture) = 7.2×10^6 pixels/s

b) Each pixel can take on one of 32 values and can therefore be represented by 5 bits:

R = 7.2×10^6 pixels/s x 5 bits/pixel = 36 Mbps

c) Allow each pixel to have one of ten intensity levels and let each pixel be one of three colors (red, blue, green) for a total of 10 x 3 = 30 levels for each pixel element.

d) We use the formula: $C = B \log_2 (1 + SNR)$

B = 4.5 x 106 MHz = bandwidth, and

SNR = 35 dB = $10^{35/10}$ = 3162, and therefore

$C = 4.5 \times 10^6 \log_2 (1 + 103.5) = 4.5 \times 10^6 \log_2 (3163) = (4.5 \times 10^6 \times 11.63) = 52.335 \times 10^6$ bps.

e) Spectral efficiency = $C/B = 52.335\times10^6/4.5\times10^6$ = 11.63 bit/sec/Hz.