KFUPM - COMPUTER ENGINEERING DEPARTMENT

COE-241 – Data and Computer Communication

Quiz 02 Due Monday March 11th, 2013 - solution key

Student Name:

Student Number:

Problem 1 (20 point) On the subject of Z-transform

Solution:

a) The Z-transform for signal $x(n)$ is given by

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty} (0.2)^n z^{-n} = \sum_{n=0}^{\infty} \left(\frac{0.2}{z}\right)^n$$
$$= \frac{1}{1-0.2/z} = \frac{z}{z-0.2}$$

b) Note that $x(n)$ for part (b) is identical to $x(n)$ defined in part (a). Therefore, $X(z)$is the same as that computed for part (a).

c) The Z-transform for signal $x(n)$ is given by

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} = 2\sum_{n=4}^{\infty} \left(\frac{0.2}{z}\right)^n = 2\left\{\left(\frac{0.2}{z}\right)^4 + \left(\frac{0.2}{z}\right)^5 + \left(\frac{0.2}{z}\right)^6 + \cdots\right\}$$
$$= 2\left(\frac{0.2}{z}\right)^4 \left\{1 + \left(\frac{0.2}{z}\right)^1 + \left(\frac{0.2}{z}\right)^2 + \cdots\right\} = 2\left(\frac{0.2}{z}\right)^4 \frac{1}{1-0.2/z}$$
$$= \frac{2(0.2)^4}{z^4(z-0.2)}$$

d) The inverse Z-transform for $X(z)$ is obtained by matching terms to the pairs given in class. Therefore,

$$x(n) = 5(0.2)^n u(n)$$

e) To find the Z-transform we must write the function X(z) using partial fraction expansion. One can show that $X(z) = \frac{6}{(z-0.8)(z-0.2)}$ may be expanded as follows

$$X(z) = \frac{20}{(z-0.4)} + \frac{-20}{(z-0.2)}$$

Also the form $1/(z-p)$ need to be written as $\frac{1}{p}\{z/(z-p)-1\}$. Therefore, $X(z)$ can be written as

$$X(z) = \frac{20}{0.4}\left\{\frac{z}{z-0.4} - 1\right\} + \frac{-20}{0.2}\left\{\frac{z}{z-0.2} - 1\right\}$$
$$= \frac{50z}{z-0.4} + \frac{-100z}{z-0.2} + 50$$

Now, by inspection, the series x(n) should be:

$$x(n) = 50(0.4)^n - 100(0.2)^n + 50\delta(n)$$

Or using Matlab –

```
>> B = [0 0 4];
>> A = conv([1 -0.2], [1 -0.4]);
>> [r, p, k] = residuez(B, A)

r =

   50.0000
 -100.0000


p =

    0.4000
    0.2000


k =

   50.0000

>>
```

conv() is used to multiply polynomials. This means that

$$X(z) = \frac{4}{(z-0.2)(z-0.4)} = \frac{50}{(1-0.4z^{-1})} + \frac{-100}{(1-0.2z^{-1})} + 50$$

Which can be inverted directly using the pairs listed in class. i.e.

$$x(n) = 50(0.4)^n - 100(0.2)^n + 50\delta(n)$$