KFUPM - COMPUTER ENGINEERING DEPARTMENT

COE-241 – Data and Computer Communication
Assignment 1 – Due date: Feb 18th, 2013

Student Name:
Student Number:

Problem #	Maximum Mark	Mark
1	10	
2	20	
3	30	
4	100	
Total	160	

Problem 1 (10 point)

Perhaps the major disadvantage is the processing and data overhead. There is processing overhead because as many as seven modules (OSI model) are invoked to move data from the application through the communications software. There is data overhead because of the appending of multiple headers to the data. Another possible disadvantage is that there must be at least one protocol standard per layer. With so many layers, it takes a long time to develop and promulgate the standards.

Problem 2 (20 point)

a) The function s(t) can be written as $s(t) = A|\cos(2\pi t/T)|$ for any $t \in \mathbb{R}$. However for the purpose of carrying out the FSE one needs to select an appropriate period. A good choice would be to select the period $t \in \left(-\frac{T}{4}, \frac{3T}{4}\right)$. For this period, the function s(t) is defined as follows

$$s(t) = \begin{cases} A\cos(2\pi t/T) & \frac{-T}{4} \le t \le \frac{T}{4} \\ -A\cos(2\pi t/T) & \frac{T}{4} \le t \le \frac{3T}{4} \end{cases}$$

The above specification contains the two branches that can be used in the integrals required to find the Fourier Series Expansion for s(t).

b) Power for s(t) is given by

$$P = \frac{1}{T}\int_0^T |s(t)|^2 dt$$

This is the same as finding the power of a regular cosine --> The power is equal to A^2/2.

Problem 3 (30 point)

a) Mathematical expression for s(t)

$$s(t) = \begin{cases} 0 & \frac{-T}{2} \le t \le 0 \\ \frac{-2A}{T}t + A & 0 \le t \le \frac{T}{2} \end{cases}$$

b) s(t) is analog; s(t) assumes all (continuous) values from A to 0 when t changes from 0 to T/2. It has a period equal to T=2 seconds. The fundamental frequency is given by $f_0 = 1/T = 0.5$ Hz.

c) The DC component is equal to average area under s(t) for one period. The area is equal to ½ x A x T/2 = AT/4 squared units. The average area is equal to the area divided by T ➔ average area = AT/4 / T = A/4 = 0.5 units (or volts).

d) Since s(t) has a DC component, then fmin = 0.

Since s(t) has points of infinite slope, then fmax = infinity

Therefore, theoretical bandwidth = fmax – fmin = infinity

e) Energy of s(t), Es is given by

$$E_s = \int_{-\infty}^{\infty} |s(t)|^2 dt = \infty$$

Power for s(t), Ps is given by

$$P_s = \frac{1}{T}\int_{-T/2}^{T/2} |s(t)|^2 dt = \frac{1}{T}\int_0^{T/2} \left|\frac{-2A}{T}t + A\right|^2 dt = \frac{1}{T}\int_0^{T/2} \left(\frac{4A^2}{T^2}t^2 - \frac{4A^2}{T}t + A^2\right) dt$$

$$= \frac{1}{T}\left[\frac{4A^2}{T^2}\frac{t^3}{3} - \frac{4A^2}{T}\frac{t^2}{2} + A^2 t\right]_0^{\frac{T}{2}} = \frac{1}{T}\left[\frac{A^2T}{6} - \frac{A^2T}{2} + \frac{A^2T}{2}\right] = \frac{A^2}{6}$$

Then Ps is equal to $A^2/6 = 4/6 = 0.667$ Watts.

Problem 4 (100 point)

a) The function s(t) can be defined as:

$$s(t) = \begin{cases} -A & -T/2 \le t \le 0 \\ A & 0 \le t \le T/2 \end{cases}$$

Other definitions are also possible.

b) s(t) is a discrete function since it assumes only two values -A and A. Period is equal to T = 1 sec. Fundamental frequency f0 = 1/T = 1 Hz.

c) The DC component for s(t) is the average area for one period. Therefore, the DC component is equal to T/2 x (-A) + T/2 x (A) = 0 Volts. OR

$$DC = \frac{1}{T}\int_{-T/2}^{T/2} s(t)dt = \frac{1}{T}\int_{-T/2}^{0} (-A)\, dt + \frac{1}{T}\int_0^{T/2} (A)\, dt = 0 \text{ volts}$$

d) fmin = f0 Hz (since it does not has a DC component), fmax = infinity (since it has points of infinite slope or sharp edges). Theoretical bandwidth for s(t) is equal to infinity.

e) The power of s(t) is given by

$$P_s = \frac{1}{T}\int_{-T/2}^{T/2} |s(t)|^2 dt = \frac{2}{T}\int_0^{T/2} A^2\, dt = A^2 = 4 \text{ Watts}$$

f) The Fourier Series expansion is given by:

$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos(2\pi n f_0 t) + B_n \sin(2\pi n f_0 t)$$

where the coefficients are given by

$$A_0 = \frac{2}{T}\int_{-T/2}^{T/2} s(t)dt$$

$$A_n = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\cos(2\pi n f_0 t)\, dt \text{, and}$$

$$B_n = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\sin(2\pi n f_0 t)\, dt$$

Therefore,

$$A_0 = \frac{2}{T}\int_{-T/2}^{T/2} s(t)dt = \frac{2}{T}\int_{-T/2}^{0} (-A)\, dt + \frac{2}{T}\int_0^{T/2} A\, dt = 0$$

$$A_n = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\cos(2\pi n f_0 t)\, dt = \frac{-2A}{T}\int_{-T/2}^{0} \cos(2\pi n f_0 t)\, dt + \frac{2A}{T}\int_0^{T/2} \cos(2\pi n f_0 t)\, dt$$

$$= \frac{-2A}{T}\int_0^{T/2} \cos(2\pi n f_0 t)\, dt + \frac{2A}{T}\int_0^{T/2} \cos(2\pi n f_0 t)\, dt$$
$$= 0$$

$$B_n = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\sin(2\pi n f_0 t)\, dt = \frac{-2A}{T}\int_{-T/2}^{0} \sin(2\pi n f_0 t)\, dt + \frac{2A}{T}\int_0^{T/2} \sin(2\pi n f_0 t)\, dt$$
$$= \frac{4A}{T}\int_0^{T/2} \sin(2\pi n f_0 t)\, dt = \frac{-4A}{2\pi n} \times \cos(2\pi n f_0 t)|_{t=0}^{t=T/2} = \frac{-2A}{\pi n}\left[\cos\left(\frac{2\pi n}{T} \times \frac{T}{2}\right) - \cos\left(\frac{2\pi n}{T} \times 0\right)\right]$$
$$= \frac{-2A}{\pi n}[\cos(\pi n) - 1] = \frac{2A}{\pi n}[1 - \cos(\pi n)] = \begin{cases} 0 & n \text{ even} \\ \frac{4A}{\pi n} & n \text{ odd} \end{cases}$$

Therefore, s(t) is given by

$$s(t) = 0 + \frac{2A}{\pi}\sum_{n=1,2,3}^{\infty} \frac{[1-\cos(\pi n)]}{n}\sin(2\pi n f_0 t)$$
$$= \frac{4A}{\pi}\sum_{n=1,3,5}^{\infty} \frac{1}{n}\sin(2\pi n f_0 t)$$

g) The power of s(t) using the FSE is equal to

$$P_{FSE} = \frac{16A^2}{2\pi^2}\sum_{n=1,3,5}^{\infty}\frac{1}{n^2} = \frac{8A^2}{\pi^2} \times \frac{\pi^2}{8} = A^2 \text{ Watts}$$

➔ Evaluating this quantity, for A = 2, one can show that P_{FSE} is equal to 4.0 Watts.

h) It is required to find k such that power for s_e(n=k) is 90% of total power of s(t).

k	**s_e(n=k)**	**fmin**	**fmax**	**BW**	**Power**	**% (relative to total power)**
1	$s\_e(n=1) = \frac{4A}{\pi}\sin(2\pi t/T)$	f_0 = 1 Hz	f_0 = 1 Hz	0 Hz	$P = \frac{16A^2}{2\pi^2} = 3.242$ Watts	81.06%
2	$s\_e(n=3) = \frac{4A}{\pi}\sin(2\pi t/T) + \frac{4A}{3\pi}\sin(2\pi t(3)/T)$	f_0 = 1 Hz	$3f_0$ = 3 Hz	$2f_0$ = 2 Hz	$P = \frac{16A^2}{2\pi^2} + \frac{16A^2}{2(3\pi)^2} = 3.603$ Watts	90.06%

To obtain 90% of the power, it is required to include up to and including n = 3.

i) The bandwidth for the new truncated series is 2 x f_0 = 2 Hz.

j) The PSD function is given by

$$PSD(f) = \frac{1}{2}\sum_{n=1,3,5}^{\infty}\frac{A^2}{(\pi n)^2}\delta(f - nf_0)$$

Note that PSD function exists only at discrete points in the frequency domain (i.e. integer multiples of f0).

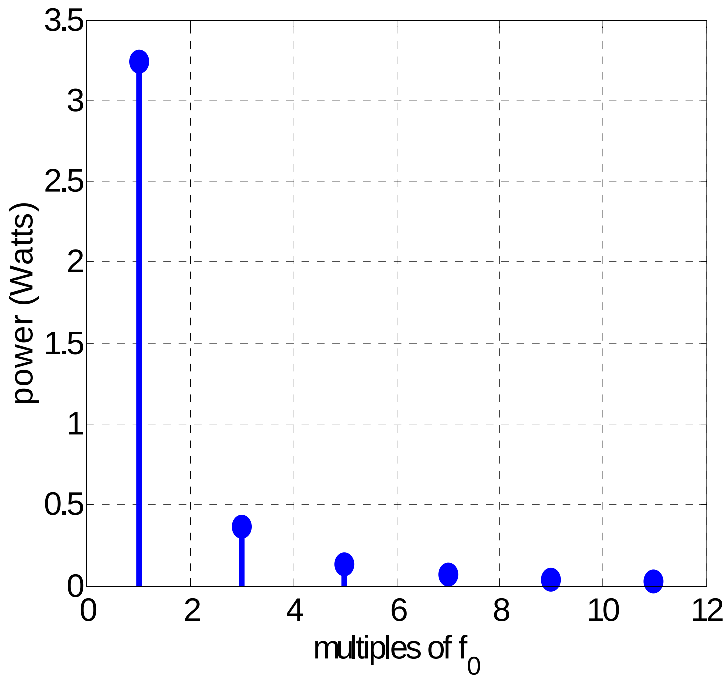

Figure: Power spectral density for s(t) – figure not required.